Sep 23

PROPOSITION: Let T be a BFS tree for $G=(V,E)$

If $(u,w) \in E$ s.t. $u \in L_i$, $w \in L_j$

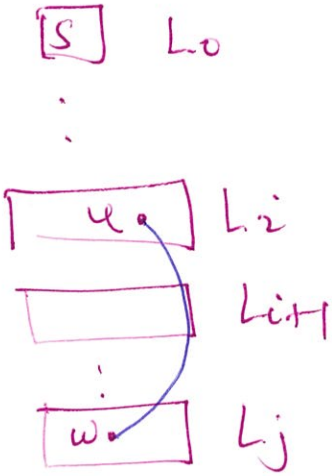
$$\Rightarrow |i-j| \le 1 \iff i \in \{j-1, j, j+1\}$$

Pf (idea): WLOG assume $i \le j$ [If $i > j$ change the roles of i & j]

WLOG → Without loss of generality

For contradiction assume $|i-j| > 1 \Rightarrow j > i+1$

$\Rightarrow j \ge i+2$



Consider the situation when BFS is creating L_{i+1}

→ $u \in L_i$, $w \notin L_0, \ldots, L_i$

→ $(u,w) \in E$

$\Rightarrow w$ will be added by BFS to L_{i+1}

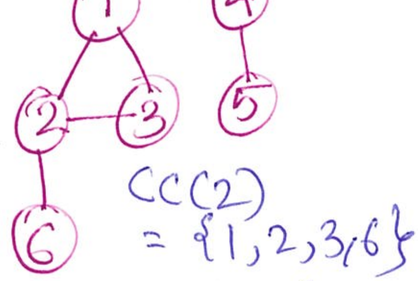
$\Rightarrow$ contradicts $w \in L_j$ for $j \ge i+2$ ∎

EXPLORE(s)

0. $R \leftarrow \{s\}$
1. While $\exists\, (u,w) \in E$ s.t. $w \notin R$, $u \in R$ (such that)

   Add w to R
2. Output $R^* \leftarrow R$

Def: Set of all vertices connected to s is called its connected component $CC(s)$



$CC(2) = \{1,2,3,6\}$

$CC(5) = \{4,5\}$

BFS