Sep 10

Gale-Shapley algorithm

⓪ Initially all men/women are free

① In a loop: (in book: men propose)
A free woman proposes to a man

② You have n matched pairs

Initial state: All n men + n women are free

① Let w be a free woman

Q: Which man m should w propose to?

A: The man m on top of L_w.
→ w proposes to m

Q2: What should m do?
Accept? X
Reject? X
→ (m, w) get engaged.

Sep 13

All men/women are either free or engaged.

① All n men & women are engaged.
→ Algo terminates & outputs the n engaged pairs.

② ∃ a free woman w

Q1) Who should w propose to?

A1: Best man m that w has not proposed to yet.
→ w proposes to m?

Q2) What should m do?

Case 1: m is free ⇒ (m, w) get engaged

Case 2: (m, w') are engaged [$w' \neq w$]

Case 2-1: $w' > w$ in L_m
⇒ nothing changes

Running Example

n = 2; M = {BP, BBT}; W = {JA, AJ}

L_{AJ}: BBT > BP
L_{JA}: BP > BBT
L_{BP}: AJ > JA
L_{BBT}: JA > AJ

AJ	JA	BP	BBT
Free	Free	Free	Free

Q: Who should JA propose to?

A: BP

→ (JA → BP) proposal

Q: What should BP do?

Accept? X
Reject? X
→ (BP, JA) get engaged

AJ	JA	BP	BBT
Free	Engaged	Engaged	Free

Q: Who should AJ propose to?

A: BBT

AJ → BBT proposal

Q: What should BBT do?

(BBT, AJ) get engaged

AJ	JA	BP	BBT
E	E	E	E

Case 2-2: $w > w'$ in L_m
⇒ (m, w) get engaged
w' is free.