

Lecture 22

CSE 331

Oct 24, 2022

Project deadlines coming up

Fri, Oct 28	Counting Inversions  F21  F19  F18  F17 x ²	[KT, Sec 5.3] (Project (Problem 1 Coding) in)
Mon, Oct 31	Multiplying large Integers  F21  F19  F18  F17 x ²	[KT, Sec 5.5] (Project (Problem 1 Reflection) in) <i>Reading Assignment: Unraveling the mystery behind the identity</i>
Wed, Nov 2	Closest Pair of Points  F21  F19  F18  F17 x ²	[KT, Sec 5.4]
Fri, Nov 4	Kickass Property Lemma  F21  F19  F18  F17 x ²	[KT, Sec 5.4] (Project (Problem 2 Coding) in)
Mon, Nov 7	Weighted Interval Scheduling  F21  F19  F17 x ²	[KT, Sec 6.1] (Project (Problem 2 Reflection) in)

Group formation instructions

Autolab group submission for CSE 331 Project

The lowdown on submitting your [project](#) (especially the [coding](#) and [reflection](#)) problems as a group on Autolab.

Follow instructions **EXACTLY** as they are stated

The instructions below are for Coding Problem 1

You will have to repeat the instructions below for EACH coding AND reflection problem on project on Autolab (with the appropriate changes to the actual problem).

Form your group on Autolab

Groups on Autolab will NOT be automatically created

You will have to form a group on Autolab by yourself (as a group). Read on for instructions on how to go about this.

Please be in touch w/ your group

note #304

stop following 64 views

Actions

Please respond to your project group mates

Please do respond back if a group project member reaches out to you to get started on the project. Just FYI, I always reserve the right to kick you out of your group (which means a 0 for you) in case you are unresponsive to repeated requests from your group members.

I understand some of you might be busy now— it is fine with me if your group decide as a whole how the work will be divided (so e.g. someone does less work on the initial problems and someone does more work on the later problems). As long as the group agrees, I do not care about the details.

But please do respond back in a timely fashion: not doing so is you not doing your part in a group project.

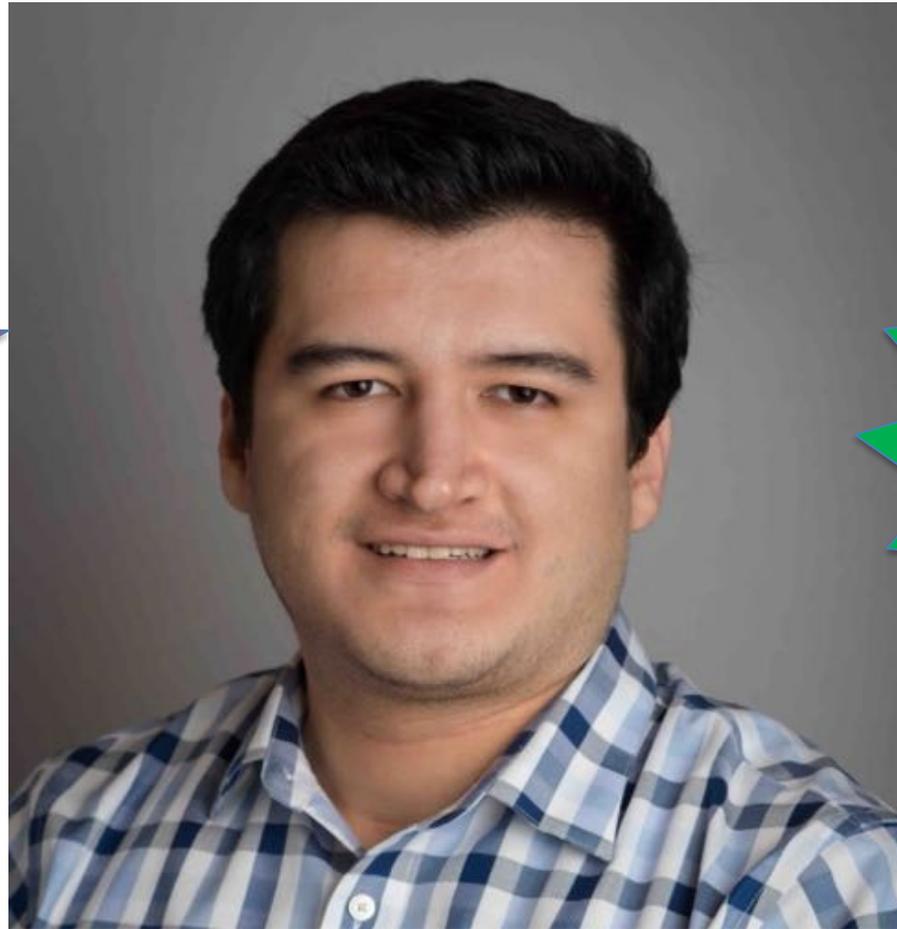
project

Edit good note 0

Updated 6 days ago by Ari Rudis

Guest lecture on Wed

My 4pm
OH
canceled



Trevor
will run
late night
OH

A. Erdem Sariyüce

One time amnesty for AI violation

note @348 stop following 1 view Actions

One time amnesty option

If for whatever reason you did not follow the [HW policies](#) for any of homeworks 1-5 (e.g. you looked at sources that you should not have, or collaborated in a group of size >3 including yourself, etc.), we're giving an option for one time amnesty. What this means is you can withdraw any question(s) where you might have violated any HW policy (and get a 0 for the corresponding question(s)) without any academic integrity violation charges.

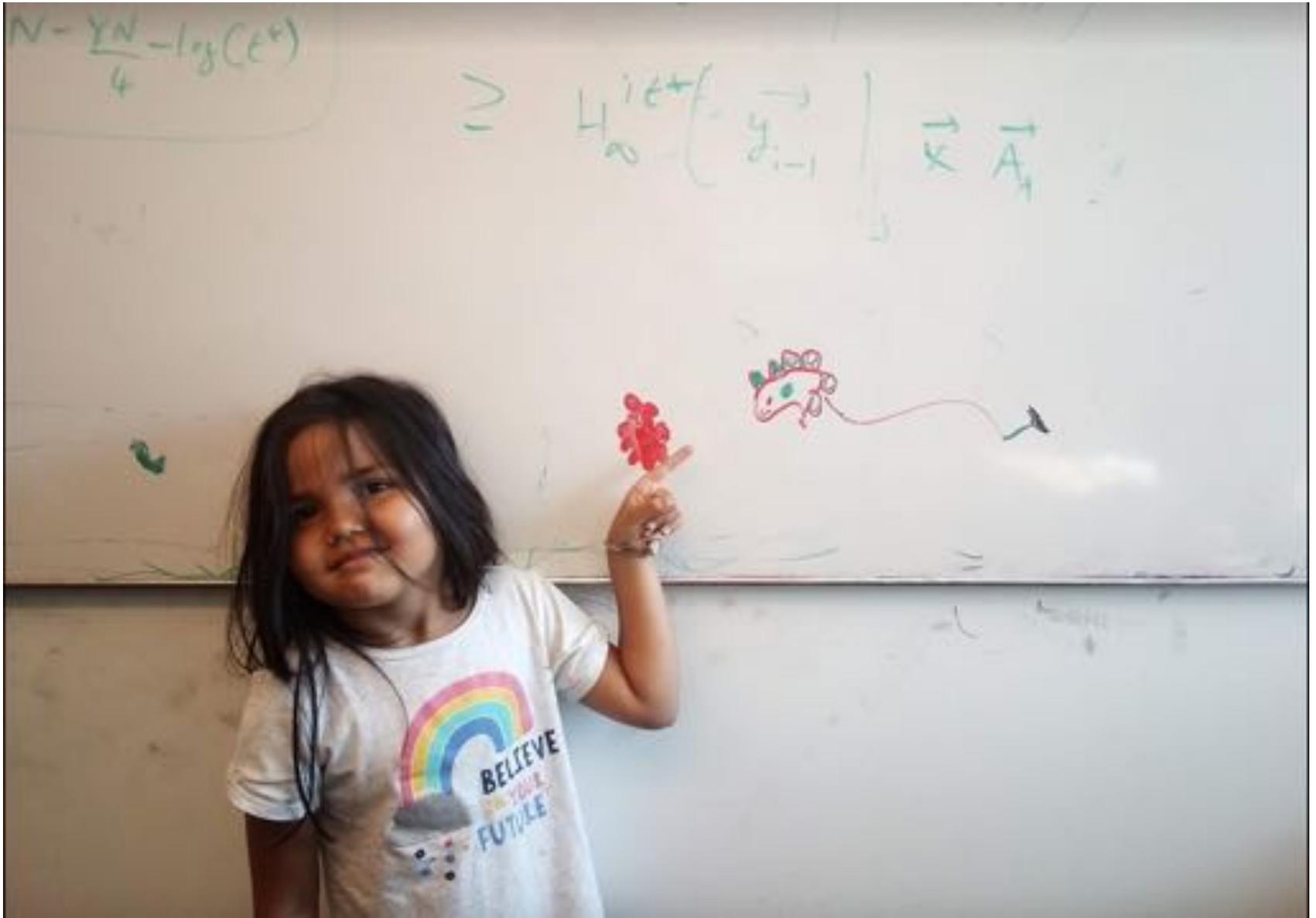
To avail of this option, **please email Atri by Thursday 5pm** letting him know which question(s) you would like to withdraw. You do NOT have to give any reason for the withdrawing the questions.

This will be the only option during the semester. If you get caught violating HW policies in HWs 1-5 after Th 5pm (or in any future HWs), then I will start academic integrity violation procedure against you.

[homework1](#) [homework2](#) [homework3](#) [homework4](#) [homework5](#) [grading](#)

[Edit](#) good note | 0 Updated 31 seconds ago by Atri Rudra

Questions/Comments?



Kruskal's Algorithm

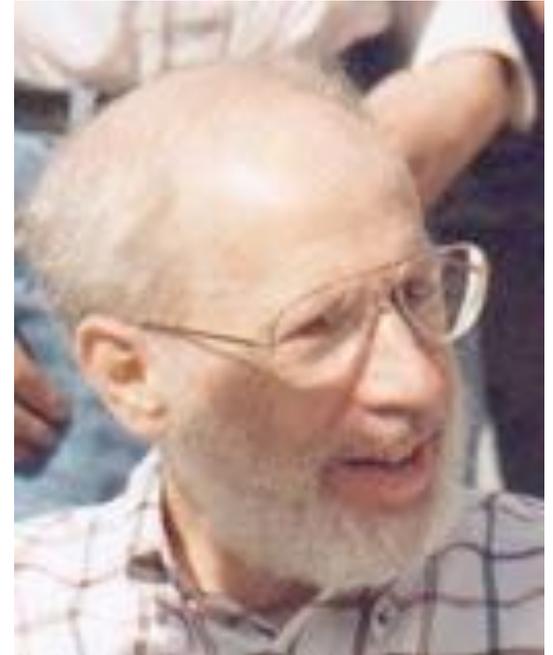
Input: $G=(V,E)$, $c_e > 0$ for every e in E

$T = \emptyset$

Sort edges in increasing order of their cost

Consider edges in sorted order

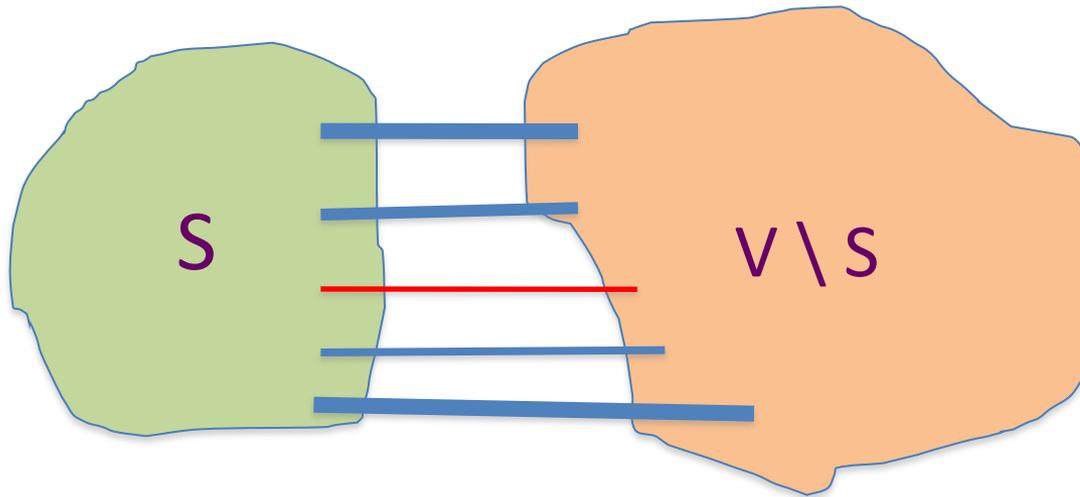
If an edge can be added to T without adding a cycle then add it to T



Joseph B. Kruskal

Cut Property Lemma for MSTs

Condition: S and $V \setminus S$ are non-empty



Cheapest crossing edge is in **all** MSTs

Assumption: All edge costs are distinct

Today's agenda

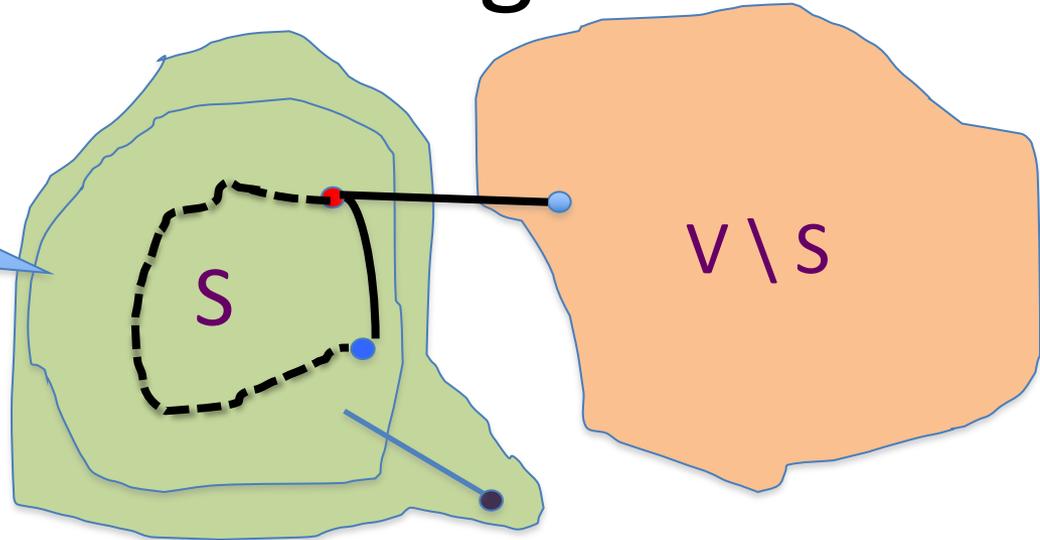
Optimality of Kruskal's algorithm

Remove distinct edge weights assumption

Quick runtime analysis of Prim's+Kruskal's

Optimality of Kruskal's Algorithm

Nodes connected to red in (V, T)



Input: $G=(V,E)$, $c_e > 0$ for every e in E

$T = \emptyset$

Sort edges in increasing order of their cost

Consider edges in sorted order

If an edge can be added to T without adding a cycle then add it to T

S is non-empty

$V \setminus S$ is non-empty

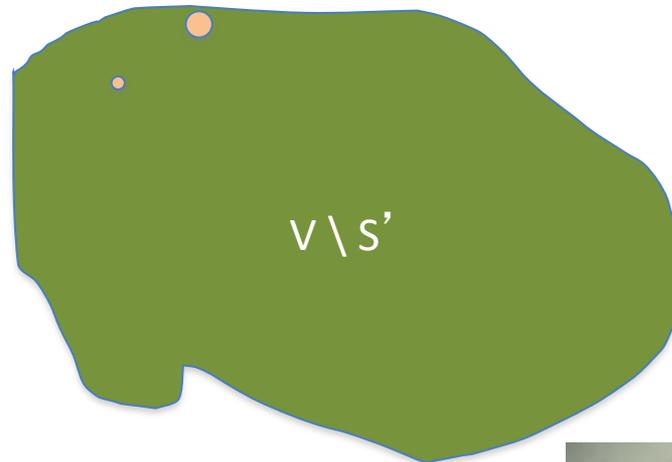
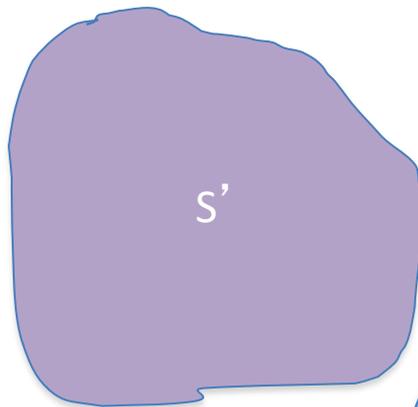
First crossing edge considered

Is (V, T) a spanning tree?

No cycles by design

Just need to show that (V, T) is connected

G is
disconnected!



No edges here



Removing distinct cost assumption

Change all edge weights by very small amounts

Make sure that all edge weights are distinct



MST for “perturbed” weights is the same as for original

Changes have to be small enough so that this holds

EXERCISE: Figure out how to change costs

Questions/Comments?



Running time for Prim's algorithm

Similar to Dijkstra's algorithm

$O(m \log n)$



Input: $G=(V,E)$, $c_e > 0$ for every e in E

$S = \{s\}$, $T = \emptyset$

While S is not the same as V

Among edges $e = (u,w)$ with u in S and w not in S , pick one with minimum cost

Add w to S , e to T

Running time for Kruskal's Algorithm

Can be implemented in $O(m \log n)$ time (Union-find DS)

Input: $G=(V,E)$, $c_e > 0$ for every e in E

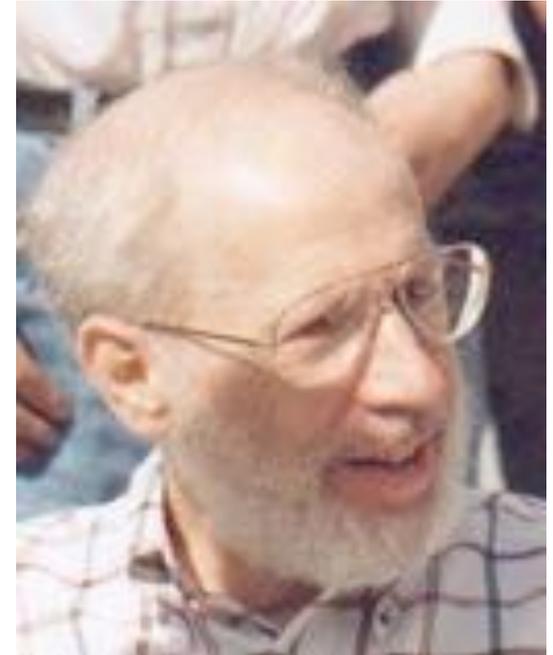
$T = \emptyset$

Sort edges in increasing order of their cost

Consider edges in sorted order

If an edge can be added to T without adding a cycle then add it to T

$O(m^2)$ time overall



Joseph B. Kruskal

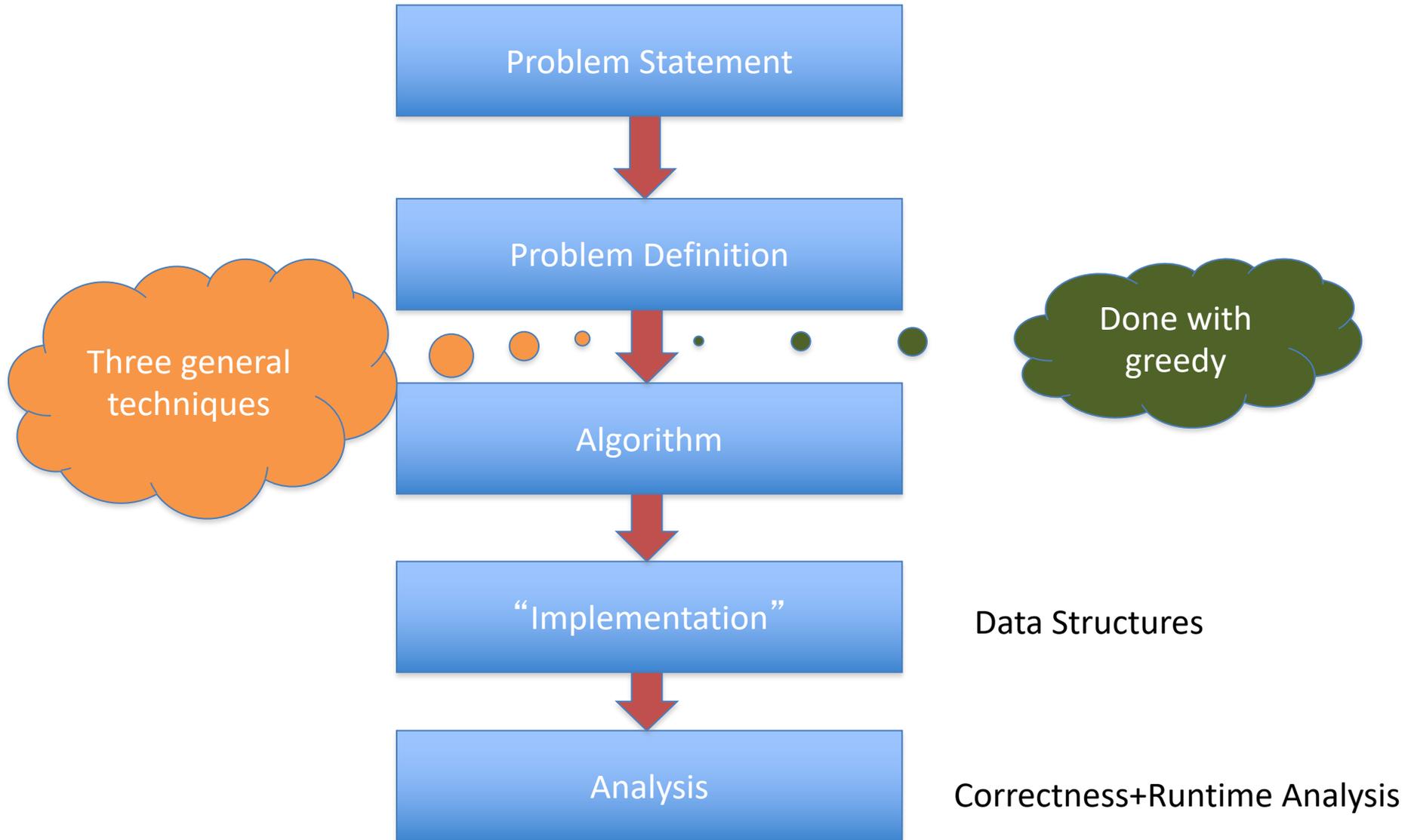
Can be verified in $O(m+n)$ time

Reading Assignment

Sec 4.5, 4.6 of [KT]



High Level view of the course



Trivia



Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

“Patch up” the solutions to the sub-problems for the final solution

Sorting

Given n numbers order them from smallest to largest

Works for any set of elements on which there is a total order

Insertion Sort

Input: a_1, a_2, \dots, a_n

Output: b_1, b_2, \dots, b_n

$O(n^2)$ overall

Make sure that all the processed numbers are sorted

$b_1 = a_1$

for $i = 2 \dots n$

Find $1 \leq j \leq i$ s.t. a_i lies between b_{j-1} and b_j

Move b_j to b_{i-1} one cell "down"

$b_j = a_i$

$O(\log n)$

$O(n)$

a	b
4	2
3	2
2	4
1	4

Other $O(n^2)$ sorting algorithms

Selection Sort: In every round pick the min among remaining numbers

Bubble sort: The smallest number “bubbles” up

Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

“Patch up” the solutions to the sub-problems for the final solution

Mergesort Algorithm

Divide up the numbers in the middle



Unless $n=2$

Sort each half recursively

Merge the two sorted halves into one sorted output

How fast can sorted arrays be merged?



Mergesort algorithm

Input: a_1, a_2, \dots, a_n

Output: Numbers in sorted order

MergeSort(a, n)

If $n = 1$ **return** the order a_1

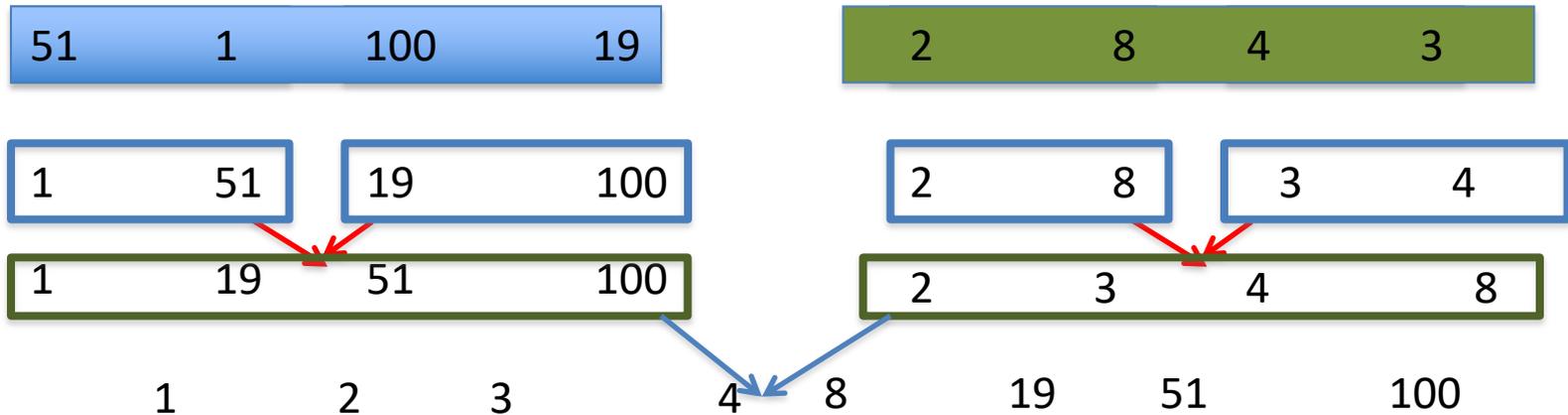
If $n = 2$ **return** the order $\min(a_1, a_2); \max(a_1, a_2)$

$a_L = a_1, \dots, a_{n/2}$

$a_R = a_{n/2+1}, \dots, a_n$

return MERGE (**MergeSort**($a_L, n/2$), **MergeSort**($a_R, n/2$))

An example run



MergeSort(a, n)

If $n = 1$ **return** the order a_1

If $n = 2$ **return** the order $\min(a_1, a_2); \max(a_1, a_2)$

$a_L = a_1, \dots, a_{n/2}$

$a_R = a_{n/2+1}, \dots, a_n$

return MERGE (**MergeSort**($a_L, n/2$), **MergeSort**($a_R, n/2$))

Correctness

Input: a_1, a_2, \dots, a_n

Output: Numbers in sorted order

MergeSort(a, n)

If $n = 1$ return the order a_1

If $n = 2$ return the order $\min(a_1, a_2); \max(a_1, a_2)$

$a_L = a_1, \dots, a_{n/2}$

$a_R = a_{n/2+1}, \dots, a_n$

return MERGE (MergeSort($a_L, n/2$) MergeSort($a_R, n/2$))

By
induction
on n

Inductive step follows from correctness of MERGE

Runtime analysis on the board...

