

Sep 26

Explore($s \in G$)

$$S = S$$

0. $R \leftarrow \{s\}$

1. While $\exists (u, w) \in E$ s.t. $w \notin R, u \in R$

Add w to R

2. Output $R^* \leftarrow R$

such that
↓



THEOREM: For all $G = (V, E)$, for all $s \in V$, $R^* = CC(s)$

If R^* is the output of $\text{Explore}(s) \Rightarrow R^* = CC(s)$

⇒ COROLLARY: BFS is correct

BFS is a special case of Explore

General idea: To show that two sets $A \neq B$ are $A = B \iff (i) A \subseteq B \text{ and } (ii) B \subseteq A$

Lemma 1: $R^* \subseteq CC(s) \iff \text{everything that is output by Explore is correct.}$

Lemma 2: $CC(s) \subseteq R^* \iff \text{everything that should be output by Explore (i.e. in } CC(s)) \text{ is actually output by Explore}$

Pf of Lemma 1: Ex (by induction)

Pf (idea) of Lem 2: Pf by contradiction

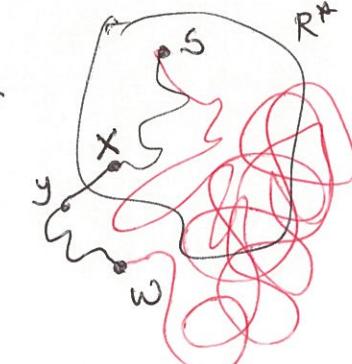
Assume $CC(s) \not\subseteq R^*$

$\Rightarrow \exists w \in CC(s) \text{ s.t. } w \notin R^*$

$\Leftrightarrow \exists \text{ an } s-w \text{ path } P \text{ in } G \text{ but } w \notin R^*$

Since P starts inside of R^* ($s \in R^*$) but ends up outside of R^* ($w \notin R^*$)

$\Rightarrow P$ has to "cross" R^* at some point



Note: Since R^* is output \Rightarrow Explore has terminated (#)

$\Rightarrow \exists (x,y) \in E$ s.t. $x \in R^*$ but $y \notin R^*$

\Rightarrow y should have been added by Explore to R

by always
defns \Rightarrow Explore should not have terminated with R^*
 \rightarrow contradicts (#)