OdA

GREEDY ALGO

$f(1) \le f(2) \le \cdots \le f(n)$

0. $R \leftarrow [n]$
1. $S \leftarrow \phi$
2. While $R \neq \phi$
   - (2.1) Let i be the smallest index in R
   - (2.2) Add i to S
   - (2.3) Remove i from R
   - (2.4) Delete all $j \in R$ that conflicts with i
3. Return $S^* \leftarrow S$

THM 1: S^* is an optimal solution

$\hookrightarrow$ $\forall$ inputs, among all possible valid schedules for that input, S^* has the max # of intervals.

Ex 1: Algo terminates.

Ex 2: S^* is a valid schedule

Pf of correctness (of greedy algos)
- $\rightarrow$ Greedy stays ahead (next)
- $\rightarrow$ Exchange argument (min. max lateness $\rightarrow$ Sec 4.2)

Pf (idea) of THM 1: Let Θ be an optimal solution

Idea 1: $S^* = \Theta$ ✗ since there can be more than 1 optimal soln

Θ

S^*

THM 2: $|S^*| = |\Theta|$ ($\Rightarrow S^*$ is also optimal)

Notation: $S^* = \{i_1, i_2, \ldots i_k\}$ $\quad f(i_1) \le f(i_2) \le \cdots \le f(i_k)$

$\Theta = \{j_1, j_2, \ldots, j_m\}$ $\quad f(j_1) \le f(j_2) \le \cdots \le f(j_k)$

THM 2': $k = m$

Claim 1: $k \le m$ (as Θ is an optimal solution)

Lemma 1: (Greedy stays ahead) $\forall \; 1 \le \ell \le k$

$$f(i_\ell) \le f(j_\ell)$$

[Assume for now Lem 1 is true]

Pf (idea) of THM 2: By contradiction

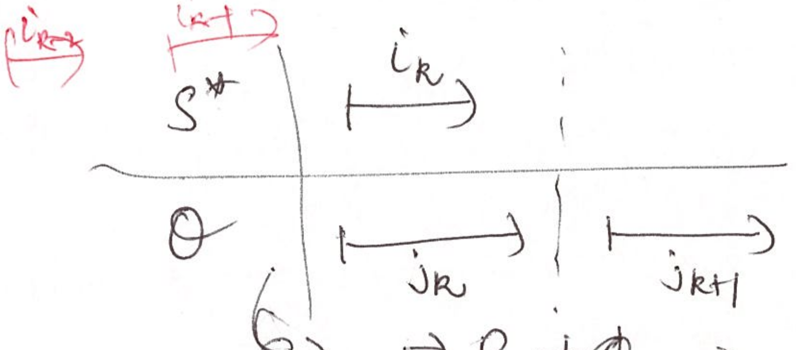
Assume $k \ne m \Longrightarrow k < m$

Claim $\Rightarrow m \ge k+1$

$\Rightarrow j_{k+1} \in \mathcal{O}$

By Lemma 1, $f(i_k) \le f(j_k)$

Consider the situation right after i_k is added to S $(= S^*)$ by greedy algo



(Algo terminates after adding i_k to S)

$\Rightarrow j_{k+1} \in R$

$\{j_{k+1}$ does NOT conflict with $i_k, i_{k-1}, i_{k-2}, \dots, i_1\}$

$\Rightarrow R \ne \emptyset \Rightarrow$ algo cannot terminate $\Rightarrow$ contradicts ∎

Pf (idea) of Lem 1 Induction on ℓ

Base case: $\ell = 1$ $\quad f(i_1) \le f(j_1)$ $\quad f(1) \le f(2) \le \dots f(n)$

$f(i_1) = f(1)$ ← defn of Greedy algo

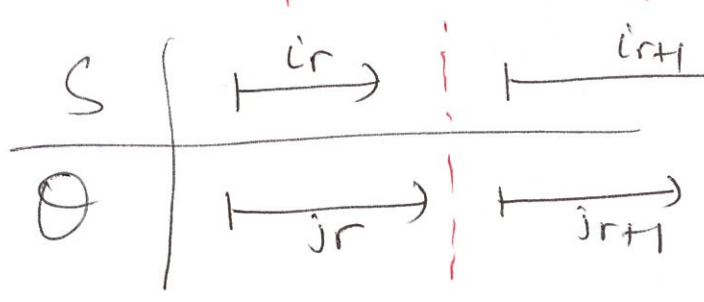
I.H Assume for some $r \ge 1$

$\forall \; 1 \le \ell \le r \quad f(i_\ell) \le f(j_\ell)$

I.S. Show $f(i_{r+1}) \le f(j_{r+1})$ ← prove by contradiction

Assume $f(i_{r+1}) > f(j_{r+1})$

Consider algo after i_r was added to S



$\Longrightarrow$ i_{r+1}, j_{r+1} $\in R$

Ex.

But as $f(j_{r+1}) < f(i_{r+1})$

$\Rightarrow$ algo should add j_{r+1} to S