Oct 2

Closest pair of points

$x_i, y_i \le \text{poly}(n)$

Input: n distinct points $p_1, \ldots, p_n$; $p_i = (x_i, y_i)$

Output: pair p_i, p_j s.t. $d(p_i, p_j)$ is min
$i \neq j$

$$d(p_i, p_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

ASSUMPTIONS:

(1) Given p_i & p_j can compute $d(p_i, p_j)$ in $O(1)$ time.

$d(p_i, p_j)$ is min $\Leftrightarrow$ $d^2(p_i, p_j)$ is min

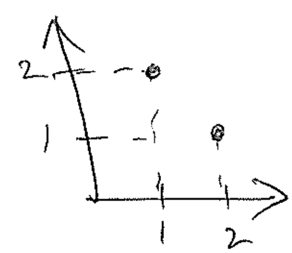
$= (x_i - x_j)^2 + (y_i - y_j)^2$

(2) Assume the x_i values are all distinct
———— y_i ————

If not (i) "rotate" all pts
(ii) Can modify the algo we'll see to handle the general case.

Notation P is the set of n pts.
$P = \{p_1, \ldots, p_n\}$

$P = \{(1,2), (2,1)\}$

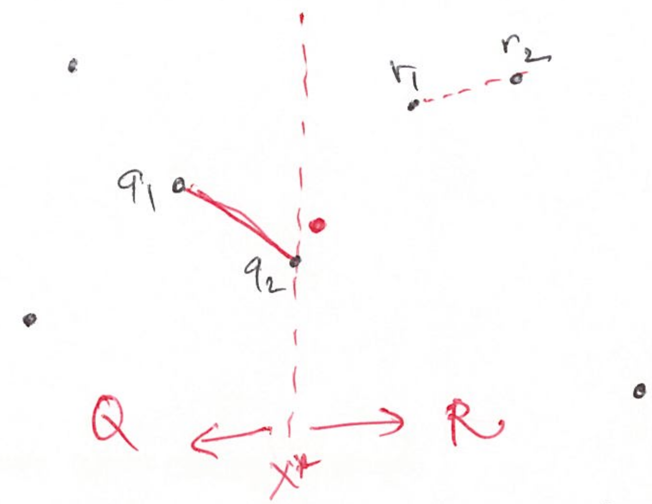


P_x : pts in P sorted by x-values $\rightarrow$ $P_x = \{(1,2), (2,1)\}$

P_y : ———— y ———— $\rightarrow$ $P_y = \{(2,1), (1,2)\}$

Towards a Divide & Conquer algo

$n = 8$



$(x^*, y^*) = P_x\left[\lceil \frac{n}{2} \rceil\right]$

$Q = \{(x,y) \in P \mid x \le x^*\}$

$R = \{(x,y) \in P \mid x > x^*\}$

Since all x co-ords are distinct $\Rightarrow |Q| = \lceil \frac{n}{2} \rceil$

$|R| = n - \lceil \frac{n}{2} \rceil = \lfloor \frac{n}{2} \rfloor$

Step 2: Recursively find q_1, q_2 the closest pair of pts in Q

——— r_1, r_2 ——— R

$\delta = \min\left(d(q_1,q_2),\ d(r_1,r_2)\right)$

ASIDE: P_x, P_y for P in $O(n \log n)$

Q: Given P_x, P_y; compute Q_x, Q_y, R_x, R_y in $O(n)$.

$\rightarrow Q_x = P_x[1 : \lceil \frac{n}{2} \rceil]$ & $R_x = P_x[\lceil \frac{n}{2} \rceil + 1 : n]$

$\rightarrow Q_y$, & R_y

$\Rightarrow$ Scan (x,y) in order of P_y, if $x \le x^*$ add (x,y) to Q_y

else add (x,y) to R_y

$O(n)$