KICKASS PROPERTY LEMMA

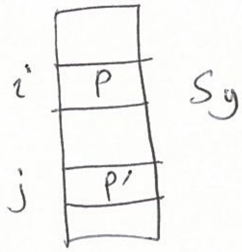
Nov 4

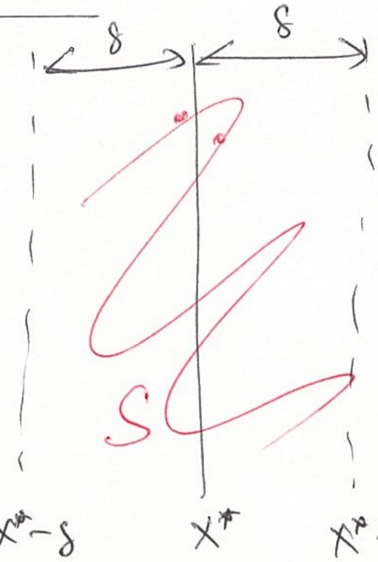
For every $p \neq p' \in S$ s.t. $d(p, p') < \delta$

If $p = S_y[i]$, $p' = S_y[j]$

then

$|i - j| \leq 15$ (!)





<u>Note:</u> Replace "15" by 9 (or even 7)

<u>Q</u>: $O(n)$ algo for Closest-in-box?

S_y pts in S sorted by y coordinates → $S = \{(x, y) \in P \mid |x - x^*| \leq \delta\}$

$x^* - \delta \leq x \leq x^* + \delta$

for $i = 1 .. n' - 1$ ← $O(n)$ time $\quad (|S| = n')$

check $(S_y[i], S_y[i+1])$, $(S_y[i], S_y[i+2])$, ... $(S_y[i], S_y[\underbrace{i+15}_{\min(i+15, n')}])$ — $O(1)$

Let (p_i, p_i') be the closest pair among the 15 above — $O(n)$

Let (p, p') be the closest pair among (p_i, p_i') $i = 1 .. n'$ — $O(n)$

If $d(p, p') < \delta$
  return (p, p')
else
  return NULL
— $O(1)$

Overall: $O(n)$

Pf (idea) of Kickass Property lemma

By contradiction

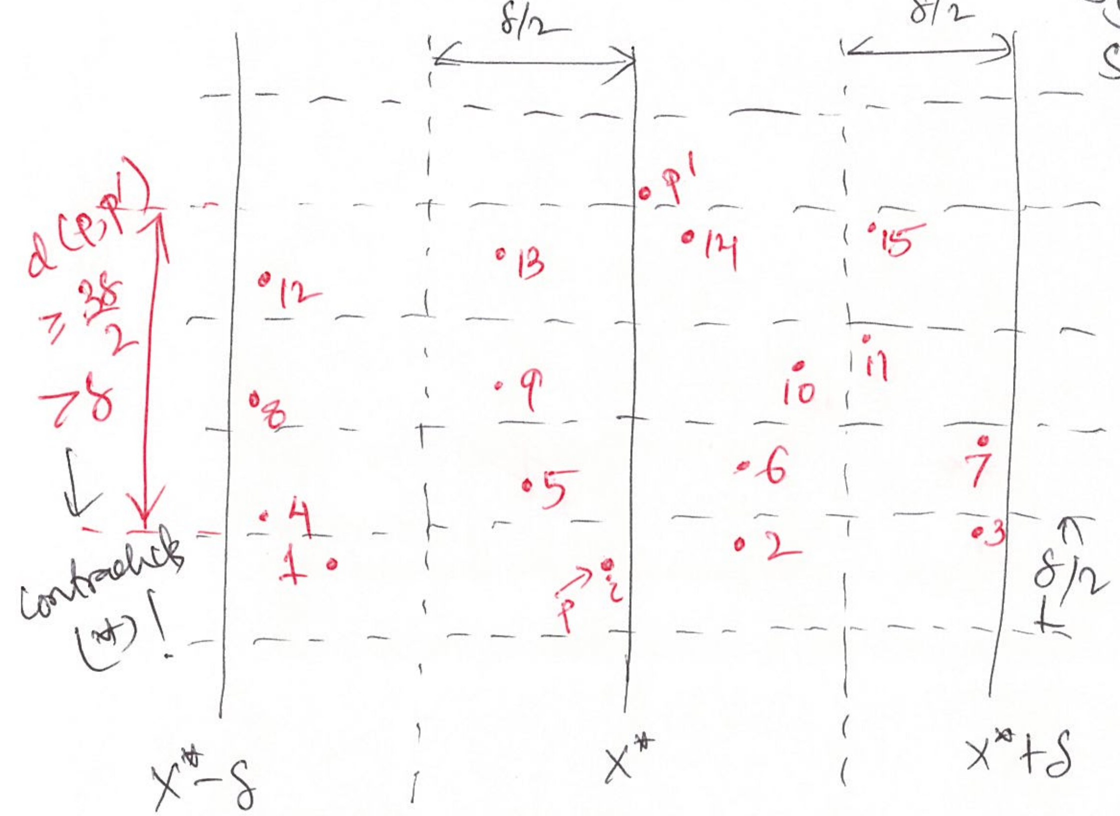
Assume $d(p, p') < \delta$ —— (*)

$S_y[i] = p$

$S_y[j] = p'$

but $j \geq i + 16$

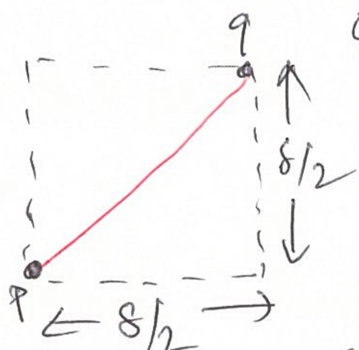
$i, i+1, \ldots, i+15, i+16$



Claim: Every $\frac{\delta}{2} \times \frac{\delta}{2}$ square has at most 1 point for S

Pf (idea) By contradiction. Assume $\exists$ p, q inside one $\frac{\delta}{2} \times \frac{\delta}{2}$ square

(Ex) p & q are furthest apart if they are on diagonal corner



$\Rightarrow d(p, q) \leq \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2}$

$= \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta$ as $\sqrt{2} > 1$

$\Rightarrow$ contradicts the definition of δ (as each square sits completely within Q or R) ☒