$\boxed{\text{Nov 11}}$

→ have access to $p(1), \ldots, p(n)$
→ Assumed $f_1 \le f_2 \le \cdots \le f_n$ } can do these in $O(n \log n)$ time

⇒ Compute $M[0..n]$

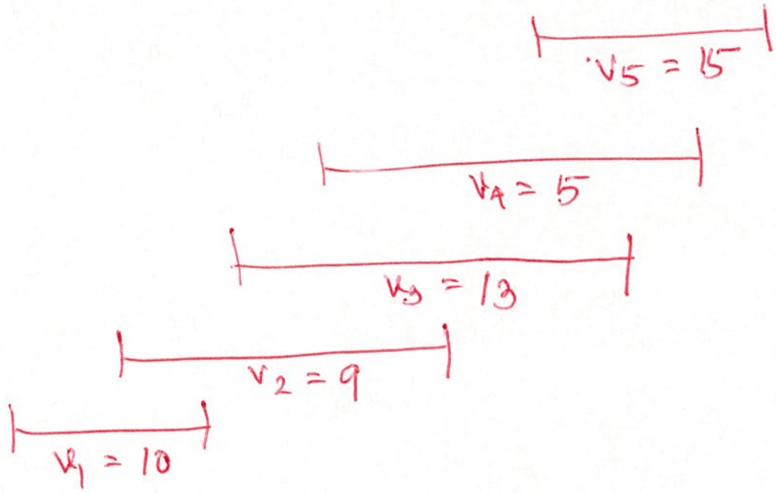
① $M[0] \leftarrow 0$

② for $j = 1..n$

$M[j] \leftarrow \max\{v_j + M[p(j)], M[j-1]\}$

③ Return $M[n]$

$\boxed{n=5}$



$p(5) = 2$

$p(4) = 1$

$p(3) = 1$

$p(2) = 0$

$p(1) = 0$

	0	1	2	3	4	5
$j=0$	0					
$j=1$	0	10				
$j=2$	0	10	10			
$j=3$	0	10	10	23		
$j=4$	0	10	10	23	23	
$j=5$	0	10	10	23	23	25

$M[0] \leftarrow 0$

$M[1] = \max\{v_1 + M[0], M[0]\}$
$= \max\{10 + 0, 0\} = 10$

$M[2] = \max\{v_2 + M[0], M[1]\}$
$= \max\{9 + 0, 10\} = 10$

$M[3] = \max\{v_3 + M[1], M[2]\}$
$= \max\{13 + 10, 10\} = 23$

$M[4] = \max\{v_4 + M[1], M[3]\}$
$= \max\{5 + 10, 23\} = 23$

$M[5] = \max\{v_5 + M[2], M[4]\}$
$= \max\{15 + 10, 23\}$
$= 25$

⇒ OPT(5) = 25

Q: What is O_5?

<u>Recall:</u> $j \in O_j$ iff $v_j + OPT(p(j)) > OPT(j-1)$

$$\underset{M[p(j)]}{\overset{\parallel}{v_j + OPT(p(j))}} > \underset{M[j-1]}{\overset{\parallel}{OPT(j-1)}}$$

can also replace by $\geq$

$\boxed{n=5}$ $5 \overset{?}{\in} O_5$ $\quad 15+10 \overset{?}{>} 23$ ✓ $\Rightarrow 5 \in O_5$

$p(5)=2$ Consider $O_5 \setminus \{5\} = O_2$ for $[2]$

$2 \overset{?}{\in} O_2$ $\quad 9+0 \overset{?}{>} 10$ X $\Rightarrow 2 \notin O_2$

Consider $O_2 = O_1$ for $[1]$

$1 \overset{?}{\in} O_1$ $\quad 10+0 \overset{?}{>} 0$ ✓ $\Rightarrow 1 \in O_1$

$\Rightarrow O_5 = \{1, 5\}$

MSchedule $(n; M, p)$

  If $n = 0$ return ϕ

  If $v_n + M[p(\cancel{j}^{\,n})] > M[\cancel{j}^{\,n}-1]$

    return $\{\cancel{j}^{\,n}\} \cup$ MSchedule $(p(n); M, p)$

  else

    return MSchedule $(\cancel{j}^{\,n}-1; M, p)$

<u>SUBSET SUM problem</u>

<u>Ex:</u> $n=3$ $\quad w_1 = 1,\ w_2 = 3,\ w_3 = 3$

Output a subset of these 3 number so that their sum is $\leq W$ & max the sum

(i) $W=7$ $\quad opt = \{1,2,3\}$

(ii) $W=6$ $\quad opt = \{2,3\}$

(iii) $W=5$ $\quad opt = \{1,2\}$ or $\{1,3\}$ $\Rightarrow$ In general the sum can be $< W$

<u>Input:</u> n integers $w_1, \ldots, w_n$; $w_i > 0$ Budget $W \geq 0$

<u>Output:</u> A subset $S \subseteq [n]$ s.t.

(i) $\sum_{i \in S} w_i \leq W$ $\quad$ (ii) maximize $w(S) = \sum_{i \in S} w_i$