Sep 27

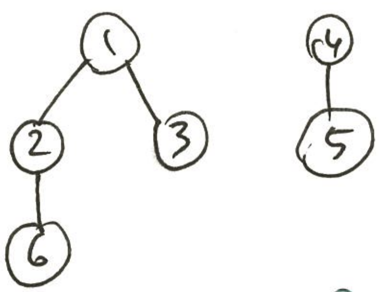
Explore (s, G)

0. $R = \{s\}$
1. While $\exists\ (u,w) \in E$ s.t. $u \in R$ and $w \notin R$

   Add w to R
2. Output $R^* = R$

Lem 0: Explore always terminates

Def: Set of all vertices connected to s is called its connected component. $CC(s)$



$CC(2) = \{1, 2, 3, 6\}$

$CC(5) = \{4, 5\}$

Theorem: For all $G = (V, E)$, start vertices s, $R^* = CC(s)$

BFS is a special case of Explore $\Rightarrow$ **Corollary:** BFS is correct

General trick: To show that two sets A & B are $A = B \iff A \subseteq B$ and $B \subseteq A$

Lemma 1: $R^* \subseteq CC(s) \rightarrow$ everything that is outputted by Explore is correct

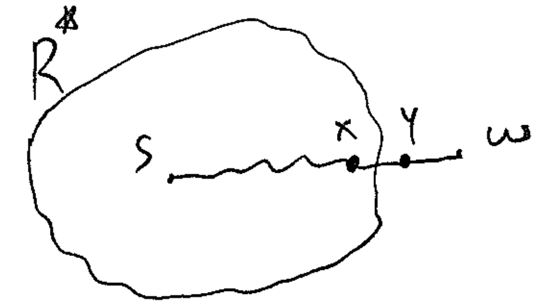
Ex. By nduction

Lemma 2: $CC(s) \subseteq R^* \rightarrow$ everything in the correct set is actually outputted by Explore

Lemma 1 + Lemma 2 $\Rightarrow$ Thm $\Rightarrow$ BFS is correct

Pf (idea) of Lemma 2 : Pf. by contradiction.

Assume $CC(s) \not\subseteq R^* \Longrightarrow \exists\, w \in CC(s)$ s.t. $w \notin R^*$



$\iff \exists$ $\underbrace{s\text{-}w \text{ path}}$ in G but $w \notin R^*$

Since p starts inside of R^* $(s \in R^*)$ but ends up outside of R^* $(w \notin R^*)$

$\Longrightarrow p$ has to cross R^* at some point

$\Longrightarrow \exists\, (x, y) \in P$ s.t. $x \in R^*$, $y \notin R^*$

(by alg defn) $\Longrightarrow y$ should have been added to R by Explore

$\Longrightarrow$ Explore should not have terminated with R

$\Longrightarrow$ contradicts with Lem 0 (with the existance of R^*) $\blacksquare$