

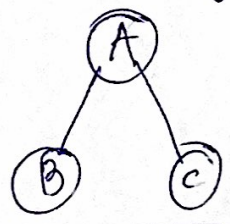
Sep 29

Lemma: $2m = \sum_{u \in V} n_u$

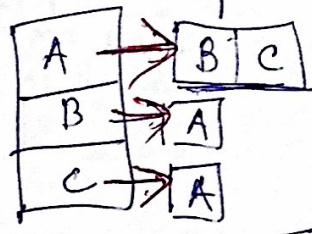
(Undirected) $G = (V, E)$

$n_u = \# \text{ neighbors of } u = |\{w \mid (u, w) \in E\}|$

$n_A = 2$
 $n_B = 1$
 $n_C = 1$



$n = 3$
 $m = 2$



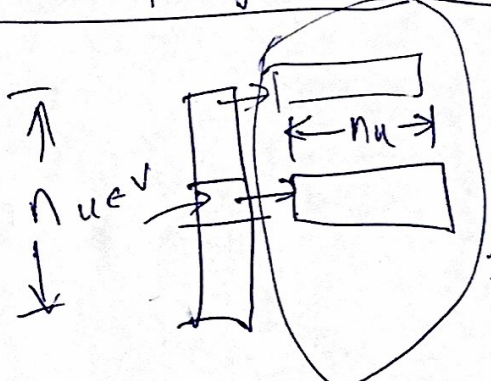
Space!
 3 pts + size of adj lists

$3 + 4 = 3 + 2 \cdot 2 = 7 = 3 + \underset{\substack{\uparrow \\ n_A}}{2} + \underset{\substack{\uparrow \\ n_B}}{1} + \underset{\substack{\uparrow \\ n_C}}{1}$

Adj. list for general G

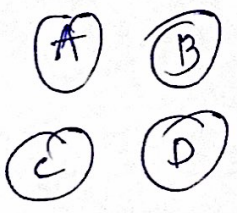
$n + 2m$

#pts = $|V| = n$



sum of list sizes = $\sum_{u \in V} n_u = 2m$

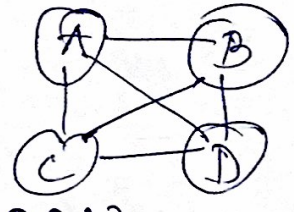
overall size = $n + 2m$



$0 \leq m \leq \binom{n}{2}$

$= \frac{n(n-1)}{2}$

$\leq \frac{n^2}{2} \leq O(n^2)$



$= \Theta(n+m) \rightarrow O(n^2)$

run BFS (G, s) // G is in adj list format

$O(n)$
cc)
1.
2.
3.

0. $CC[s] \leftarrow T$ and $CC[u] \leftarrow F \forall u \neq s \in V$

1. $i \leftarrow 0$

2. $L_0 \leftarrow \{s\}$

3. while $L_i \neq \emptyset$

\leftarrow empty set

3.1 $L_{i+1} \leftarrow \{ \}$ // $O(1)$

3.2 for $u \in L_i$

for $(u, w) \in E$

$\left[\begin{array}{l} \text{if } CC[w] = F \\ CC[w] \leftarrow T \\ \text{Add } w \text{ to } L_{i+1} \end{array} \right] O(1)$

How many times does this block run?
 $\leq T$ times

3.3 $i \leftarrow i+1$ // $O(1)$

4. Return CC // $CC(s) = \{w \mid CC[w] = T\}$
 $\uparrow O(n)$ \uparrow such that

Overall: $O(n) + T \cdot O(1)$

$= O(n) + O(T) = O(n+T)$

$T \leq O(m) \Rightarrow T \leq O(m)$ {see book}
 $\Rightarrow O(m+n)$