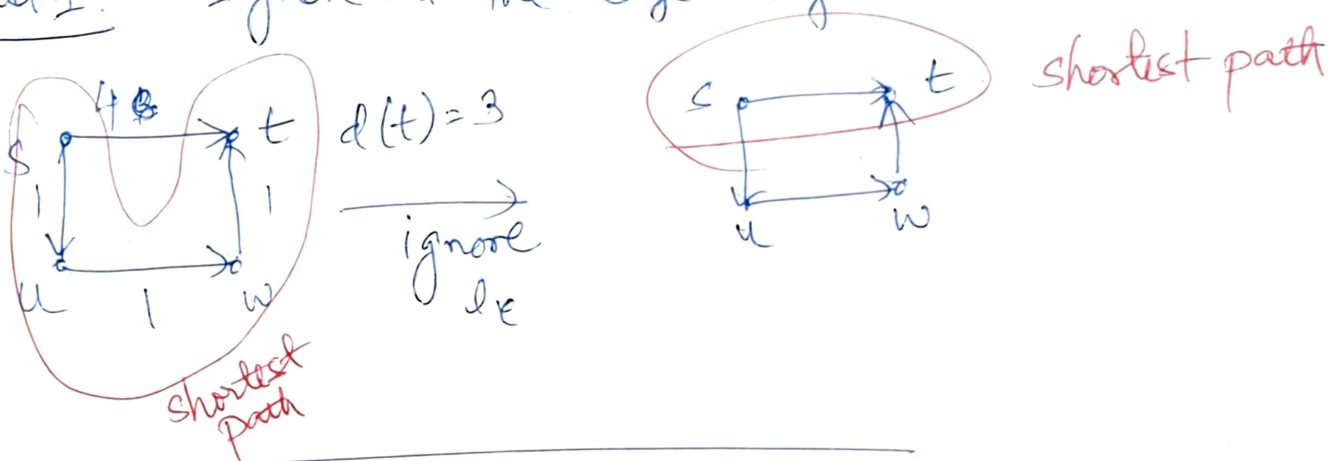
$\boxed{\text{Oct 13}}$ Special case: $l_e = L \quad \forall e \in E$

↳ Use HW3 Q3 algo (basically ignore l_e)

↳ multiply $d(t)$ from ↓ by L

Algo idea: Reduce the general case ($l_e \geq 0 \quad \forall e \in E$) to the case of $l'_e = 1$.

Idea 1: Ignore all the edge lengths

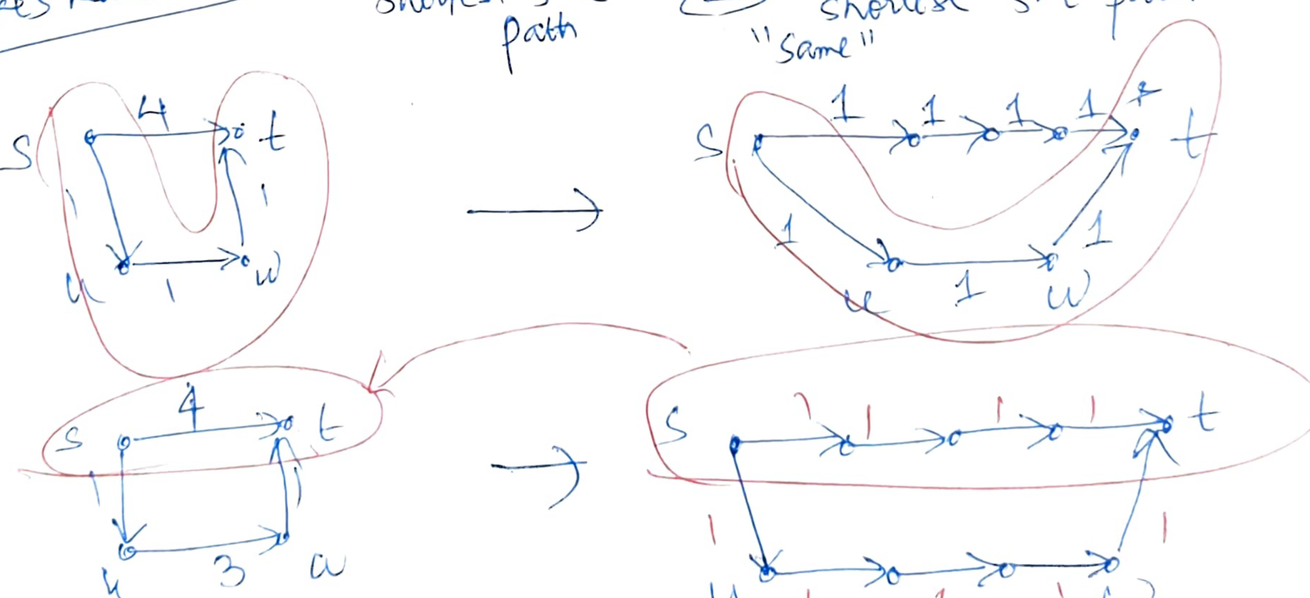


Algo idea: $G = (V, E) \longrightarrow G' = (V', E')$

$\{l_e\}_{e \in E}$ $\qquad$ $\{l'_{e'}\}_{e' \in E}$ $= 1$

Luke's reduction: shortest s-t path $\Longleftrightarrow$ shortest s-t path

"same"



$G \to G'$ (i) replace each edge $e \in E$ by
$\{l_e\}$ $\{l'_{e'}\}$ a path of length l_e in G'
$e \in E$ 1 $e' \in E$ (new paths do not share any edges)

$\to$ Run HW3 Q3 algo on G'

↳ If P' is the shortest s-t path in G'

↳ return the corresponding s-t path P in G

$G \to G' = (V', E')$

runtime of HW3 Q3 algo $= O(|V'| + |E'|)$

$n' = |V'|$, $m' = |E'|$

<u>Ex!</u> Convert $G \to G'$ in $O(n'+m')$ time

Convert $P' \to P$ in $O(n'+m')$ time

<u>OVERVALL:</u> $O(n'+m')$

Q: How does this relate to $O(n+m)$?

$n' \le l_{max} \cdot n$ $\qquad l_{max} = \max_{e \in E} l_e$

$m' \le l_{max} \cdot m$

$\Rightarrow O(l_{max} \cdot n + l_{max} \cdot m) = O(l_{max}(n+m))$ $O(1)$

If $l_{max} = O(1) \Rightarrow$ ✓ $\to$ $O(1)$ register as long as $l_{max} = n$

If $l_{max} = n^{100} \Rightarrow O(n^{100}(n+m))$ ~

<u>Aside RAM model</u> unit of space is a register

If you have n items in input, each register $O(\log n)$ bits

$\Rightarrow$ Adjacency list $O(n+m)$ registers

~~Q: How~~ Q: How many register would you need to represent $\le n^{100}$

~~A:~~ Q': How many bits $\approx 100 \log n$ $\approx O(\log n)$

$\Rightarrow \approx 100$ registers $= O(1)$ registers.