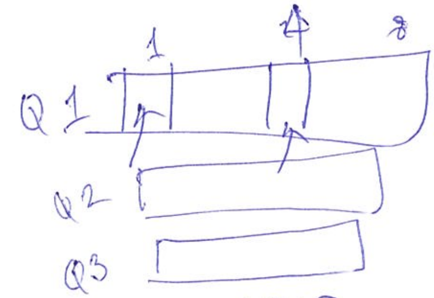
Aug 30

Prove: $\boxed{\text{Option } 2 \geq \text{Option } 1}$

Notation: Input scores: $Q1[1..8]$

$Q2[1..8]$

$Q3[1..8]$



$H[1..8]$ s.t. $\forall\ 1 \leq i \leq 8 \quad H[i] = Q1[i] + Q2[i] + Q3[i]$

$$O_1 = \underset{\text{max } 6}{\text{sum of}} \left\{ H[i] \right\}$$

$$O_2 = \text{sum of max } 6 \{ Q1[i] \} + \text{sum of max } 6 \{ Q2[i] \} + \text{sum of max } 6 \{ Q3[i] \}$$

$\boxed{\text{Theorem}}$ $O_2 \geq O_1$.

Pf: Details

Let $i_1, i_2, i_3, i_4, i_5, i_6$ $[i_1, .., i_6]$

such that

be s.t. $H[i_1], \ldots, H[i_6]$ are the max 6 HW scores.

By definition,

$$O_1 = H[i_1] + H[i_2] + \cdots + H[i_6]$$

$$= \begin{matrix} Q1[i_1] \\ + \\ Q2[i_1] \\ + \\ Q3[i_1] \end{matrix} + \begin{matrix} Q1[i_2] \\ + \\ Q2[i_2] \\ + \\ Q3[i_2] \end{matrix} + \cdots + \begin{matrix} Q1[i_6] \\ + \\ Q2[i_6] \\ + \\ Q3[i_6] \end{matrix}$$

$$= \left(Q1[i_1] + Q1[i_2] + \cdots Q1[i_6]\right) + \left(Q2[i_1] + \cdots + Q2[i_6]\right) + \left(Q3[i_1] + \cdots + Q3[i_6]\right)$$

$\binom{8}{6} = 4 \times 7 = 28$ choices of subsets of size 6 Q1 score

$$\leq \text{sum of max } 6 \; \{Q1[i]\}_{i=1..8} + \text{sum of max } 6 \; \{Q2[i]\} + \text{sum of max } 6 \; \{Q3[i]\} = \Theta_2 \Rightarrow \Theta_1 \leq \Theta_2 \quad \blacksquare$$

Proof idea: Θ_2 has more options to choose among the ~~top~~ max 6 Q1, max 6 Q2, max 6 Q3.

In particular if we pick the choices for Q1/Q2/Q3 scores from the top 6 HW scores, then each such choice is no more that the best choice for top 6 scores for Q1/Q2/Q3.