Nov 29

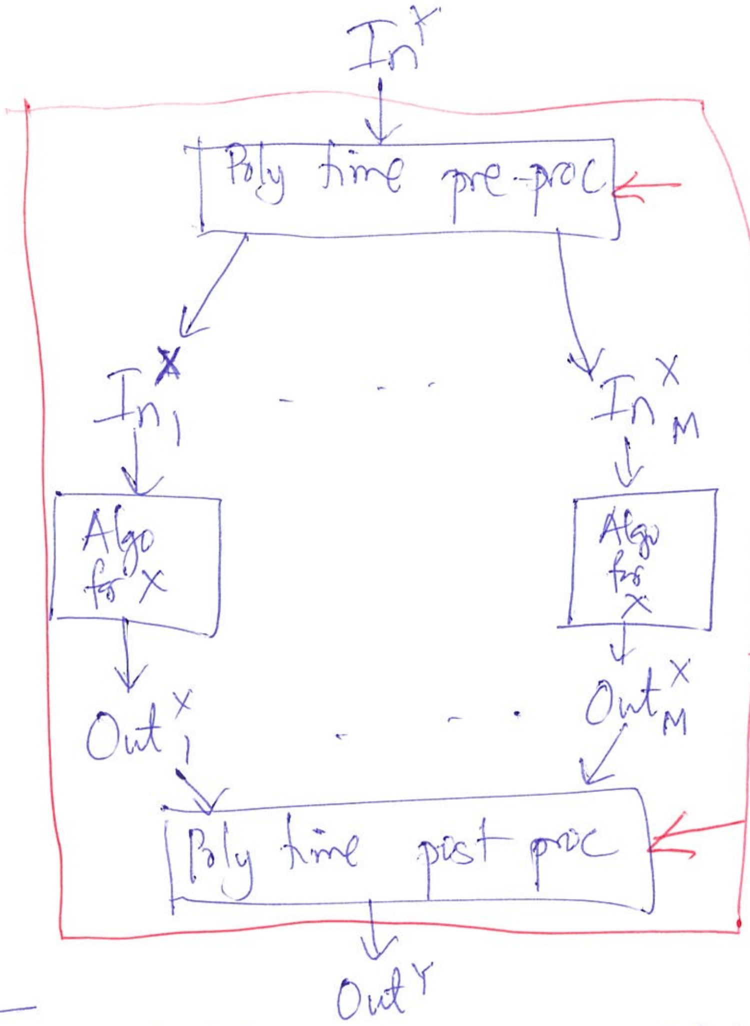
$$\underline{Y} \le_p X$$

→ Y is poly time reducible to X

≡ ∃ a poly time reduction from Y to X

Solve: $In^Y \; \cdots \rightarrow Out^Y$

Input size $N = |In^Y|$



$M \ge 1$

Usually $M = 1$

Total runtime

$= \text{poly}(N)$

$+ M \cdot (\text{Runtime} ~~\text{for}~~ \text{of Algo} ~~\text{for}~~ \text{for } X)$

$+ \text{poly}(N)$

Assume: $M \le \text{poly}(N)$

If Algo for X has poly (N) runtime

⟹ Overall

$= \text{poly}(N) + M \cdot \text{poly}(N) + \text{poly}(N)$ ← poly(N)

$\le \text{poly}(N)$

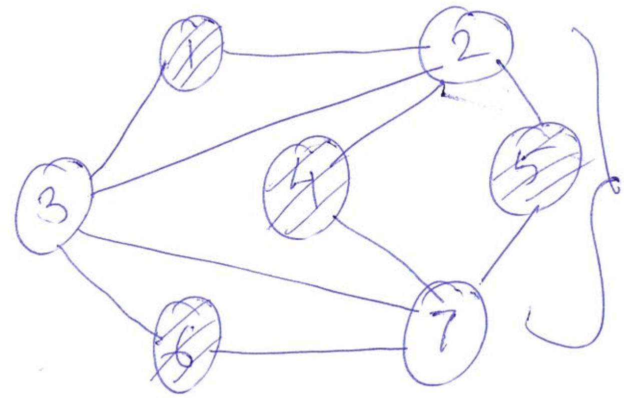
Example: HW 2 Q2 $\le_p$ Stable Matching $(M=1)$

Going forward: ONLY consider problem with Boolean output

Problem: Independent Set (IS) problem

$G = (V, E)$

Def: An IS of G is a subset $S \subseteq V$ s.t there is NO edge between any two vertices $u, w \in S$



G_0

$\{1,4\}$ ✓

$\{3,7\}$ X $\{1,4,7\}$ X

$\{3,4,5\}$ ✓

$\{1,4,5,6\}$ ✓ $n = |V|$

Formal problem — "Decision" problem

Input: $G = (V, E)$, $0 \le k \le n$

Output: TRUE/1 iff $\exists$ an IS of size $\ge k$ in G.

Ex: G_0; 2 ✓ G_0; 3 ✓ G_0; 4 ✓ G_0; 5 X It gets harder to get an IS as k increases.

Note: Any subset of an IS is also an IS

Problem 2: Vertex Cover

$G = (V, E)$. Def A vertex cover (VC) is a $C \subseteq V$ s.t ALL edges have at least one end-point in C

Ex: G_0 $\{1,2,3,4,5,6,7\}$ ✓ $\{1,2,3,4,5,6\}$ ✓

$\{1,2,6,7\}$ ✓ $\{2,3,7\}$ ✓ $\{1,7\}$ X $\{1\}$ X

Formal problem: Input: $G = (V, E)$ $0 \le k \le n$

Output: TRUE iff G has a VC of size $\le k$

Example: G_0; 6 ✓ G_0; 4 ✓ G_0; 3 ✓ G_0; 2 X

<u>Thm</u>: (1) IS $\leq_p$ VC (2) VC $\leq_p$ IS

<u>Lemma</u>: Let $G = (V, E)$. $S \subseteq V$ is an IS $\iff$ $V \setminus S$ is a VC