Sep 12

Implementing GS

Initialization $\leq T_0$

While (...) $\leq T_1$ #iterations $\leq n^2$

Body $\leq T_2$

Output S $\leq T_3$

via Lemma 0

If we show: $T_0, T_3 \leq O(n^2)$, $T_2 \leq O(1)$

Overall runtime

$\leq T_0 + T_1 \cdot T_2 + T_3$

$\leq T_0 + n^2 \cdot T_2 + T_3$

$\leq O(n^2) + n^2 \cdot O(1) + O(n^2)$

$\leq O(n^2) + O(n^2) + O(n^2)$

$\leq O(n^2)$

Goal: $T_0, T_3 \leq O(n^2)$, $T_2 \leq O(1)$ { choose the data structures }

Notation change

$W = \{w_1, \ldots, w_n\} \longmapsto \{1, \ldots, n\}$ def. $= [n]$

$M = [n]$ $\quad M = \{m_1, \ldots, m_n\} \longmapsto \{1, \ldots, n\}$

Our solution to Q1(a) on HW0

$W = [n]$ → Array indices start 1

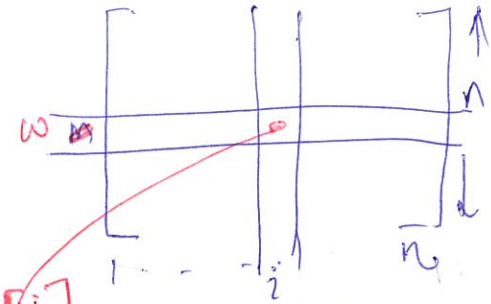
(Q0) How is the input represented?

2D arrays: WomanPref, ManPref

$n \times n$

ManPref[m][i] = ID of the i^{th} most preferred woman for m in L_m

WomanPref[w][i] = man for w in L_w



Init: N/a Query: Read WomanPref[w][i] $O(1)$ Update: n/a

(Q1) How do we find a free woman?

(A1) Maintain a linked list called free (of free women)

Init: Add all n women to free $O(n)$

Query: Pick (say) the 1st element in free (= w) ($O(1)$) (+ delete w from free)

Update: w proposes to m

Case 1: m is free ⇒ do nothing

Case 2.1 (m, w') remain engaged ⇒ Add w to free

Case 2.2 (m, w) get engaged ⇒ Add w' to free

} O(1)

(Q2) How would w pick her best unproposed man?

(A2) Maintain an array of size n called next

Next[w] = rank of the man m that w should propose to next

(Q): What is the id of the next man w should propose to?

Read [WomanPref[w][next[w]]] } O(1) time

Init next[w] = 1 ∀ w ∈ [n] } O(n) time

Query: O(1) time

Update: next[w] = next[w] + 1 O(1) time.

(Q3) How do we know who m is currently engaged to?

(A3) Array of size n call current

$$current[m] = \begin{cases} -1 & \text{if } m \text{ is free} \\ w & \text{if } (m, w) \text{ are engaged} \end{cases}$$

Init: current[m] = -1 ∀ m O(n)

Query: Read current[m] O(1)

Update: If after w's proposal, (m, w) get engaged O(1)

→ current[m] = w

Init (1) + (3) $O(n) + O(n) + O(n) = O(n) \leq O(n^2)$

Query/update $O(1) + O(1) + O(1) \leq O(1)$.