

























































Lecture 25

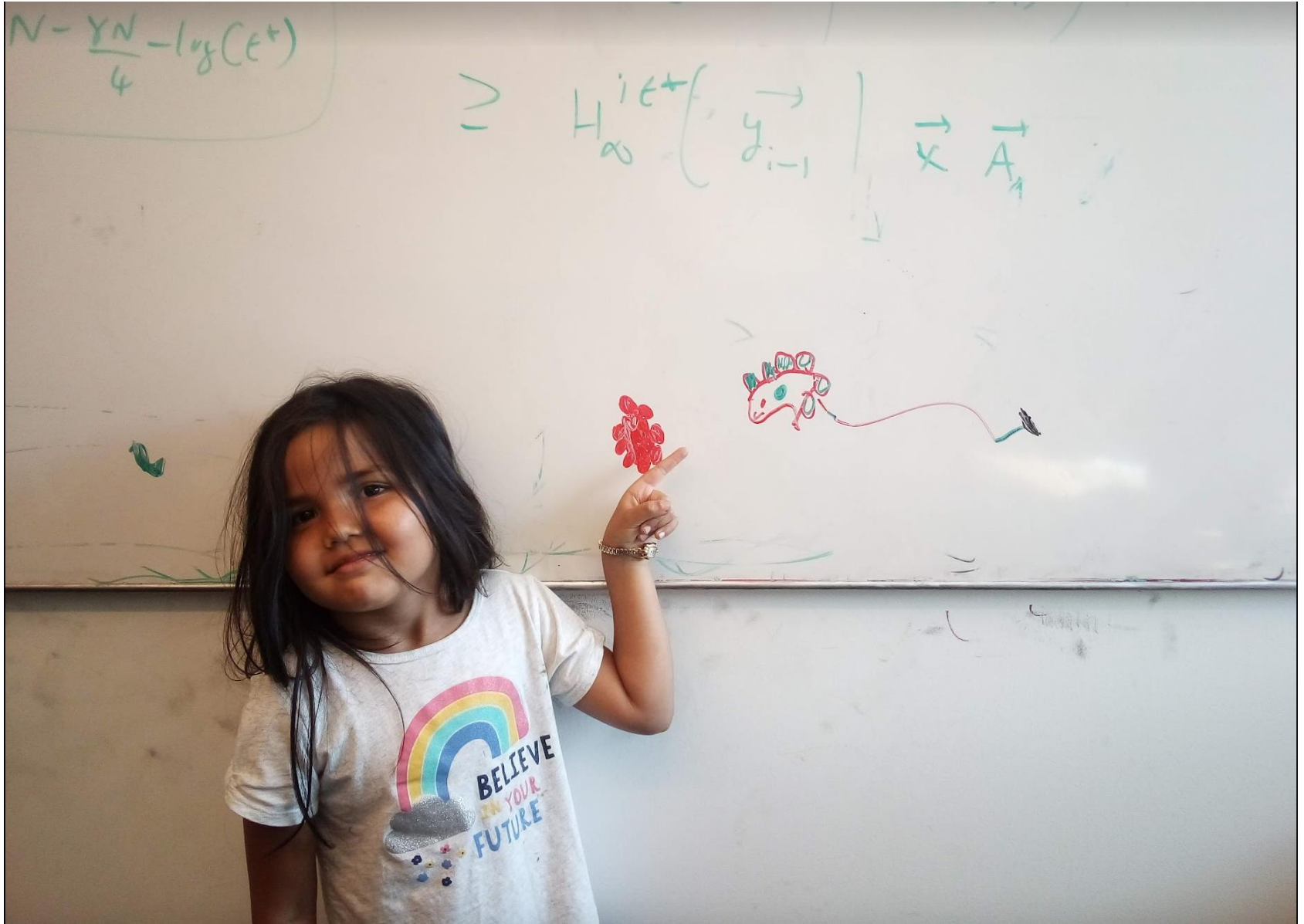
CSE 331

Oct 30, 2024

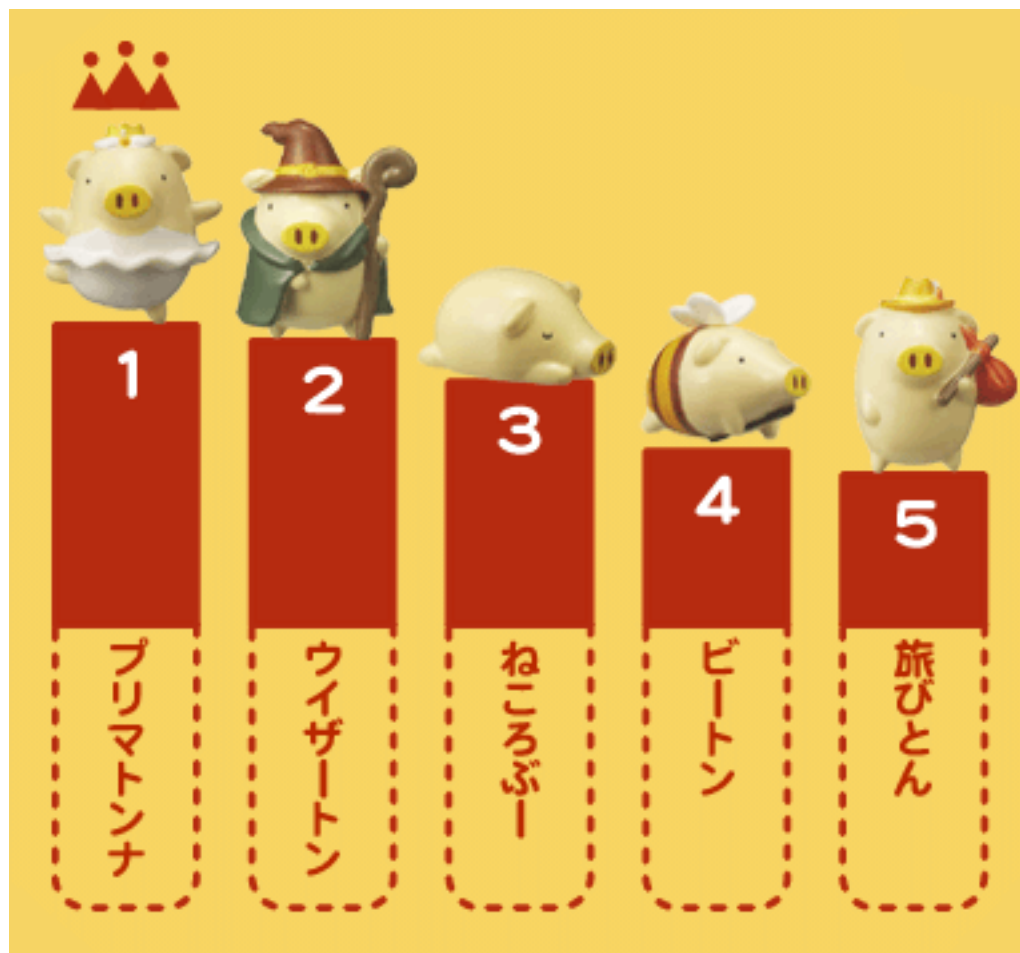
1st project deadline THIS Friday

Tue, Oct 15		(HW 4 out)
Wed, Oct 16	Dijkstra's algorithm     F24  F23  F22  F21  F19 \times^2	[KT, Sec 4.4] Week 8 recitation notes
Fri, Oct 18	Correctness of Dijkstra's Algorithm  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 4.4] <i>Reading Assignment:</i> [KT, Sec 4.4]
Mon, Oct 21	Minimum Spanning Tree  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 4.5]
Tue, Oct 22		(HW 4 in, HW 5 out)
Wed, Oct 23	Cut Property Lemma  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 4.5] <i>Reading Assignment:</i> [KT, Sec 4.5, 4.6]
Fri, Oct 25	Mergesort  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 5.1]
Mon, Oct 28	Solving recurrence relations  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 5.1]
Tue, Oct 29		(HW 5 in)
Wed, Oct 30	Counting Inversions  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 5.3]
Fri, Nov 1	Multiplying large integers  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 5.5] (Project (Problems 1 & 2 Coding) in) <i>Reading Assignment:</i> Unraveling the mystery behind the identity
Mon, Nov 4	Closest Pair of Points  F23  F22  F21  F19  F18  F17 \times^2	[KT, Sec 5.4] (Project (Problems 1 & 2 Reflection) in)

Questions/Comments?



Rankings



How close are two rankings?

Google [Advanced Search](#)

Web [Show options...](#) Results 1 - 10 of about 23,700,000 for compare rankings. (0.30 seconds)

Comparison Reviews Sponsored Link
[Angleslist.com](#) Your neighbors' ratings on local service companies. Award-winning.

Ranking - Wikipedia, the free encyclopedia
In competition **ranking**, items that **compare** equal receive the same **ranking** number, and then a gap is left in the **ranking** numbers. The number of **ranking** ...
[en.wikipedia.org/wiki/Ranking](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

A Comparison of Ranking Methods for Classification Algorithm Selection
is based on Spearman's rank correlation coefficient. To **compare ranking** methods, a combination of Friedman's test and Dunn's multiple com- ...
[www.springerlink.com/index/mh87p44744x7Q258.pdf](#) - [Similar](#) - [Print](#) - [Share](#)
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College Football Ranking Comparison
College Football **Ranking** Comparison. Sunday, November 8, 2009 (127 **Rankings**). | Last Week | Archived | FBS | FCS | Summary | CSV Data | ...
[www.masseyratings.com/cf/compare.htm](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

College Basketball Ranking Comparison
Monday, April 6, 2009 (36 **Rankings**) Cor 923 810 923 898 819 868 888 777 839 899 880
243 246 245 248 241 251 247 249 248 244 **Ranking** Violation % 254 ...
[www.masseyratings.com/cb/compare.htm](#) - [Cached](#) - [Print](#) - [Share](#)

FIFA.com - Compare Teams
With FIFA.com you can **compare** the progress of up to four teams in the FIFA/Coca-Cola World **Ranking**. See how teams have risen and fallen since August 1993 ...
[www.fifa.com/worldfootball/ranking/compare/compareteams.html](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

GetEducated.com | Rate, Rank & Compare Online Colleges & Degrees
GetEducated.com's directory of online degrees provides **rankings**, ratings, and comparison tools to help you choose the best online degrees from the best ...
[www.geteducated.com/](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

Compare your website rankings in Google Caffeine « Advanced Web ...
Aug 18, 2009 ... To help you **compare** your website **rankings** from Google with the **rankings** from Google Caffeine we have created a quick step-by-step tutorial

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ALL RESULTS 1-10 of 8,810,000 results · [Advanced](#)

FIFA.com - Compare Teams
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[www.fifa.com/worldfootball/ranking/compare/compareteams.html](#) - [Cached page](#)

How to Compare Graduate School Rankings | eHow.com
Choosing a graduate school can be a stressful and confusing process for any student. By comparing graduate schools and their **rankings** a student can choose which one fits her ...
By eHow Education Editor · Difficulty: Moderate · 0 posts
[www.ehow.com/how_2051516_compare-graduate-school-rankings.html](#) - [Cached page](#)

Ranking - Wikipedia, the free encyclopedia
Items that **compare** equal receive the same **ranking** number, which is the mean of what they would have under ordinal **rankings**. Equivalently, the **ranking** number of 1 plus the number of ...
[en.wikipedia.org/wiki/Ranking](#) - [Enhanced view](#)

Ranking Compare
Ask Google Yahoo Search | Web browser tool -- Shows search results for a given keyword or phrase on the top three engines, Google, Yahoo and Msn.
[tools.essential-seo.com/ranking-compare](#) - [Cached page](#)

SchoolDigger.com - School Rankings, Reviews and More - Public and ...
Find the best elementary, middle, and high schools. Search for schools near any address, **compare** test scores, sort by school **ranking**, class sizes, and more using SchoolDigger.
[www.schooldigger.com](#) - [Cached page](#)

College and university rankings - Wikipedia, the free encyclopedia
The College and university **rankings** are a lists of universities and liberal arts colleges in higher education, an order determined by any combination of factors.
[International ...](#) · [Regional and national ...](#) · [Criticism \(North America\)](#)
[en.wikipedia.org/wiki/College_and_university_rankings](#) - [Enhanced view](#)

www.hospitalcompare.hhs.gov
[www.hospitalcompare.hhs.gov/Hospital/Search/SearchCriteria.asp?version=default&brower=IE...](#)

Rest of today's agenda

Formal problem: Counting inversions

Divide and Conquer algorithm

Problem definition on the board...



Solve a harder problem

Input: a_1, \dots, a_n

Output: LIST of all inversions

```
L =  $\phi$ 
for i in 1 to n-1
  for j in i+1 to n
    If  $a_i > a_j$ 
      add (i,j) to L
return L
```



Optimal for
the listing
problem

Example 1: All inversions-- $(2i-1, 2i)$

2	1	3	4	6	5	7	8
---	---	---	---	---	---	---	---

Only check $(i, i+1)$ pairs

Q1: Solve listing problem in $O(n)$ time?

Q2: Recursive divide and conquer algorithm to count the number of inversions?

CountInv (a, n)

if $n = 1$ return 0

if $n = 2$ return $a_1 > a_2$

$a_L = a_1, \dots, a_{\lfloor n/2 \rfloor}$

$a_R = a_{\lfloor n/2 \rfloor + 1}, \dots, a_n$

return CountInv($a_L, \lfloor n/2 \rfloor$) + CountInv($a_R, n - \lfloor n/2 \rfloor$)

Can be horribly wrong in general

```
CountInv (a,n)
```

```
  if n = 1 return 0
```

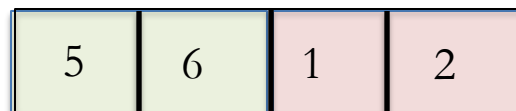
```
  if n = 2 return  $a_1 > a_2$ 
```

```
   $a_L = a_1, \dots, a_{\lfloor n/2 \rfloor}$ 
```

```
   $a_R = a_{\lfloor n/2 \rfloor + 1}, \dots, a_n$ 
```

```
  return CountInv( $a_L, \lfloor n/2 \rfloor$ ) + CountInv( $a_R, n - \lfloor n/2 \rfloor$ )
```

Example where instance has non-zero (can be $\Omega(n^2)$) inversions and algo returns 0?



All 4 “crossing” pairs are inversions

Bad case: “crossing inversions”

CountInv (a,n)

if $n = 1$ return 0

if $n = 2$ return $a_1 > a_2$

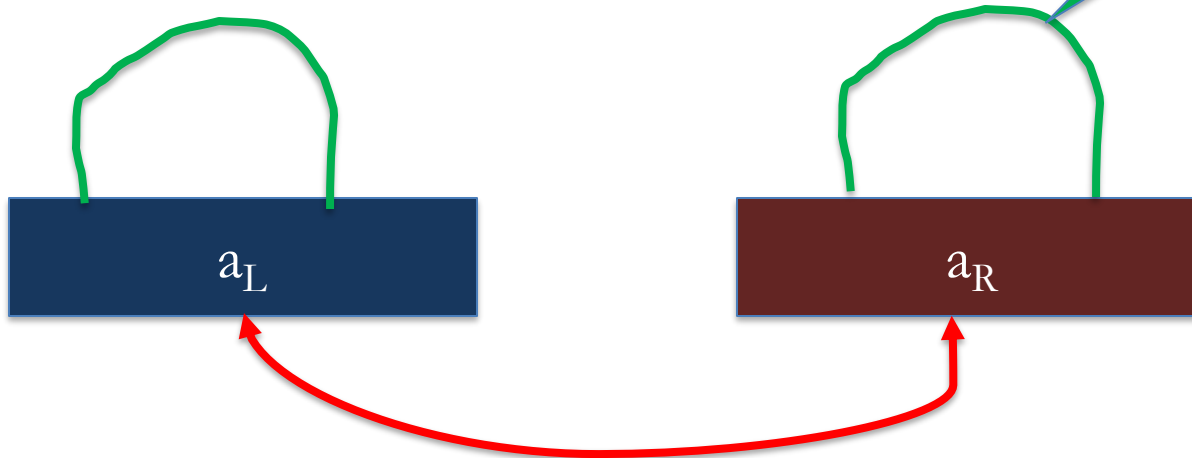
$a_L = a_1, \dots, a_{\lfloor n/2 \rfloor}$

$a_R = a_{\lfloor n/2 \rfloor + 1}, \dots, a_n$

return CountInv($a_L, \lfloor n/2 \rfloor$) + CountInv($a_R, n - \lfloor n/2 \rfloor$)

Yes!

Are a_L
and a_R
sorted?



Example 2: Solving the bad case



a_L



a_R

a_L is sorted

First element in a_L is larger than first/only element in a_R

$O(1)$ algorithm to count number of inversions?

return size of a_L

Example 3: Solving the bad case

1

a_L

5 | 6 |

a_R

a_R is sorted

First/only element is a_L is smaller than first element in a_R

$O(1)$ algorithm to count number of inversions?

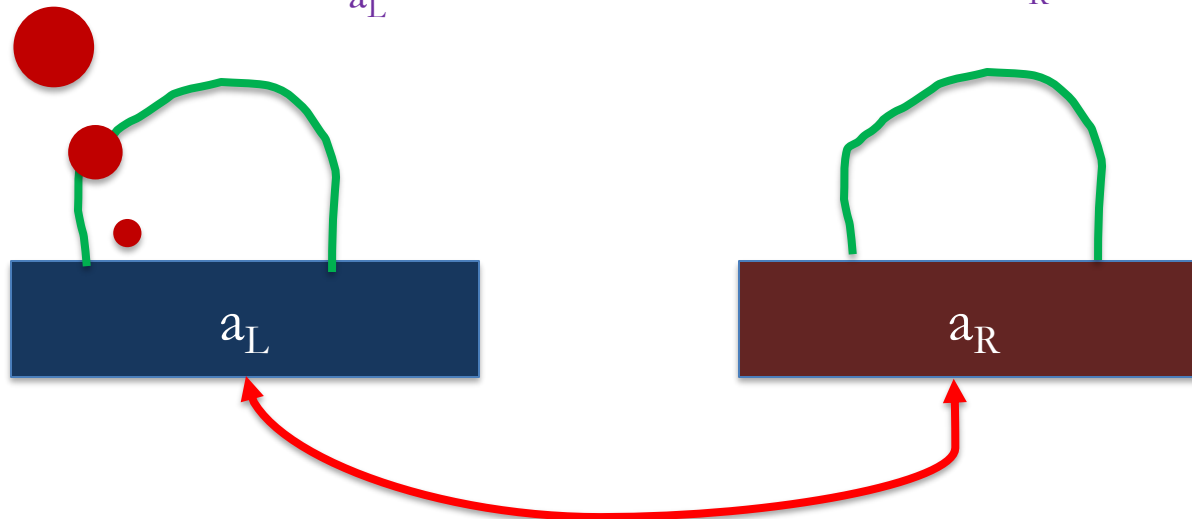
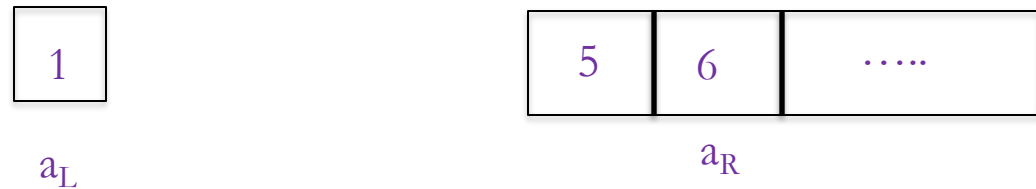
return 0

Solving the bad case

First element of a_L is larger than first element of a_R



First element of a_L is smaller than first element of a_R



Try to
modify the
MERGE
algorithm

Divide and Conquer

Divide up the problem into at least two sub-problems

Solve all sub-problems: Mergesort

Recursively solve the sub-problems

Solve stronger sub-problems: Inversions

“Patch up” the solutions to the sub-problems for the final solution

MergeSortCount algorithm

Input: a_1, a_2, \dots, a_n

Output: Numbers in sorted order+ #inversion

MergeSortCount(a, n)

If $n = 1$ return (0 , a_1)

If $n = 2$ return ($a_1 > a_2$, min(a_1, a_2); max(a_1, a_2))

$a_L = a_1, \dots, a_{n/2}$ $a_R = a_{n/2+1}, \dots, a_n$

(c_L, a_L) = MergeSortCount($a_L, n/2$)

(c_R, a_R) = MergeSortCount($a_R, n/2$)

(c, a) = MERGE-COUNT(a_L, a_R)

return ($c+c_L+c_R, a$)

$$T(2) = c$$

$$T(n) = 2T(n/2) + cn$$

$O(n \log n)$ time

$O(n)$

Counts #crossing-inversions+
MERGE

MERGE-COUNT(a_L, a_R)

$a_L = l_1, \dots, l_n$

$a_R = r_1, \dots, r_m$

$c = 0$

$i, j = 1$

while $i \leq n$ and $j \leq m$

if $l_i \leq r_j$

add l_i to output

$i++$

else

add r_j to output

$j++$

$c += n - i + 1$

Output any remaining items

return c



a_L



a_R



a_L



a_R