

Lecture 27

CSE 331

Nov 4, 2024

Reflection P1+2 due TODAY

Fri, Nov 1 Multiplying large integers  F23  F22  F21  F19  F18  F17 x^2

[KT, Sec 5.5] (**Project (Problems 1 & 2 Coding)** in)
Reading Assignment: [Unraveling the mystery behind the identity](#)

Mon, Nov 4 Closest Pair of Points  F23  F22  F21  F19  F18  F17 x^2

[KT, Sec 5.4] (**Project (Problems 1 & 2 Reflection)** in)

Final exam conflict

note @274

stop following **0 views**

Actions ▾

Final exam conflicts

I know some of you have an exam conflict with CSE 331 final exam. Since I'm not sure if I know the exact set of students with conflict, I figured I'll do a piazza post.

If you have an exam conflict with the CSE 331 final please EMAIL me by 5pm on Friday, Nov 15. If you email me after this deadline, I cannot promise to be able to give you a makeup option that works with your schedule.

Please note that the makeup final will be on *Monday, Dec 16* (i.e. a day before the scheduled final exam). My goal is to pick a time that works for everyone on Dec 16.

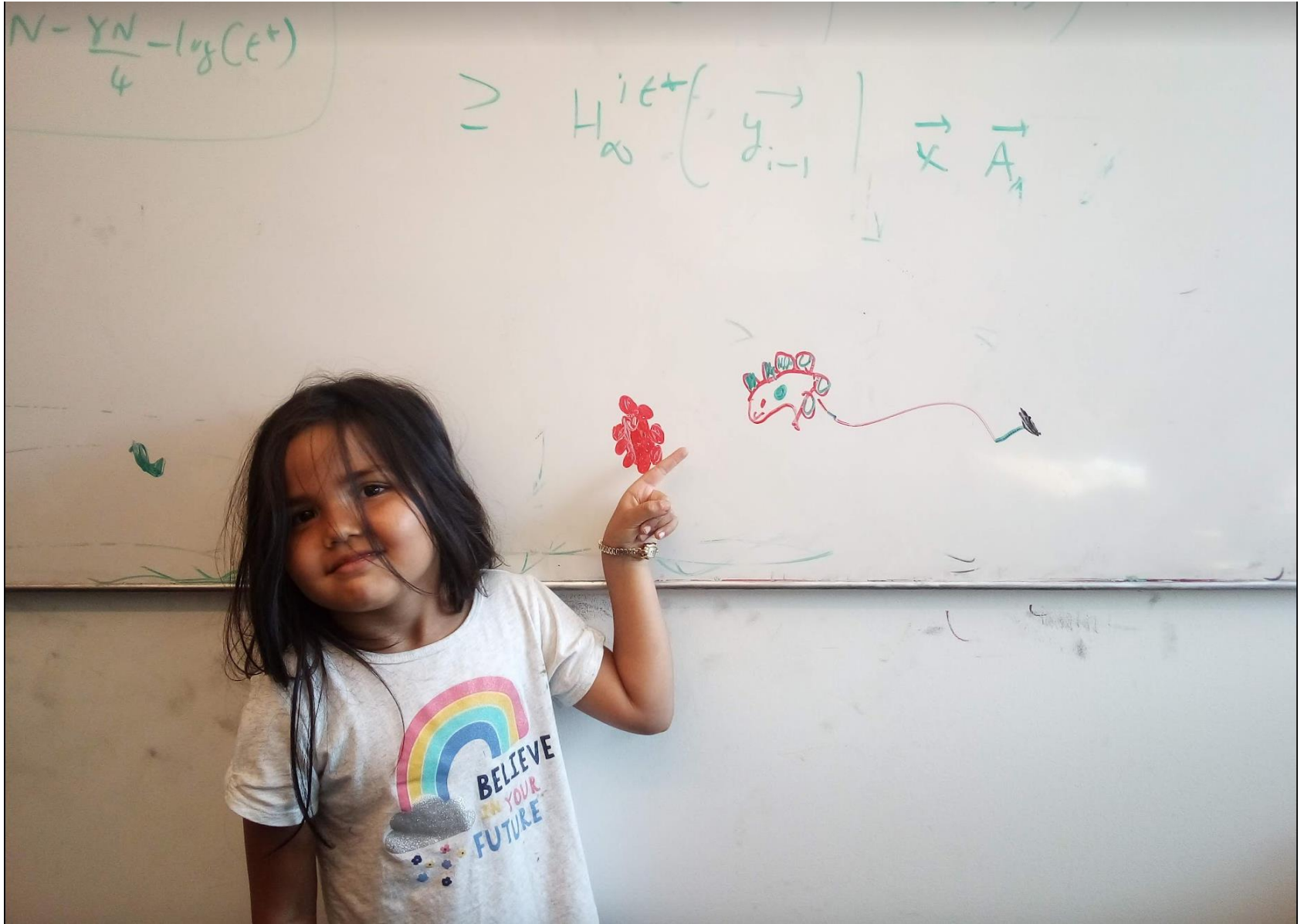
So if you email me for a makeup final exam, please send me all the time(s) that you do a makeup on Monday, Dec 16 between 9am-5pm.

final

Edit good note | 0

Updated 41 seconds ago by Atri Rudra

Questions/Comments?

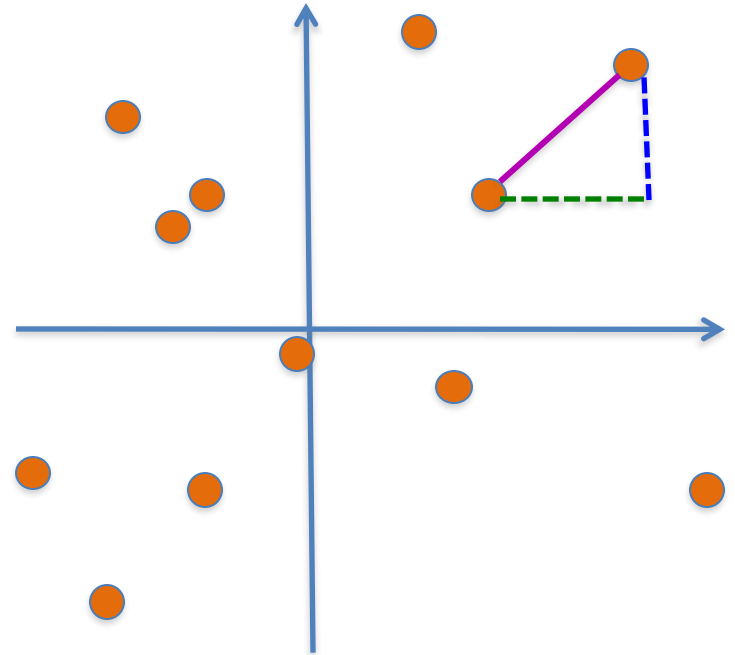


Closest pairs of points

Input: n 2-D points $P = \{p_1, \dots, p_n\}$; $p_i = (x_i, y_i)$

$$d(p_i, p_j) = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}$$

Output: Points p and q that are closest



Group Talk time

$O(n^2)$ time algorithm?

1-D problem in time $O(n \log n)$?

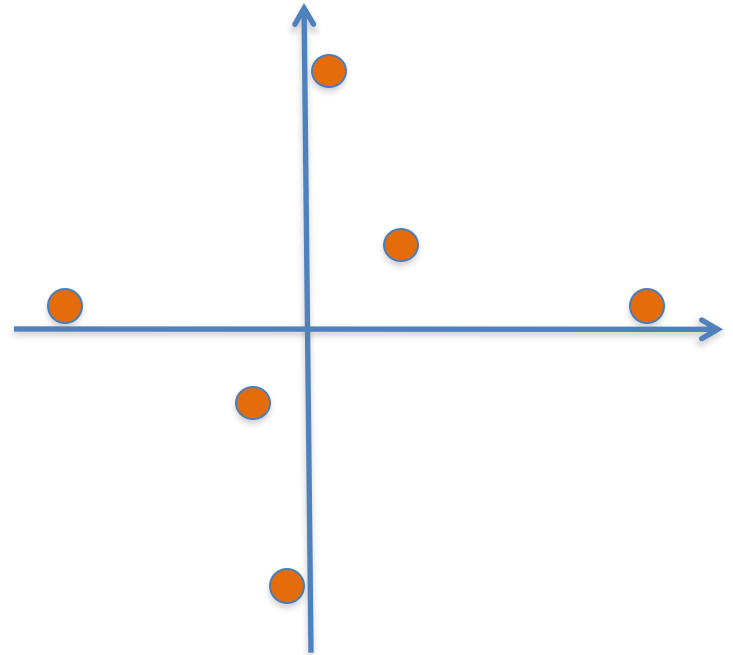


Sorting to rescue in 2-D?

Pick pairs of points closest in **x** co-ordinate

Pick pairs of points closest in **y** co-ordinate

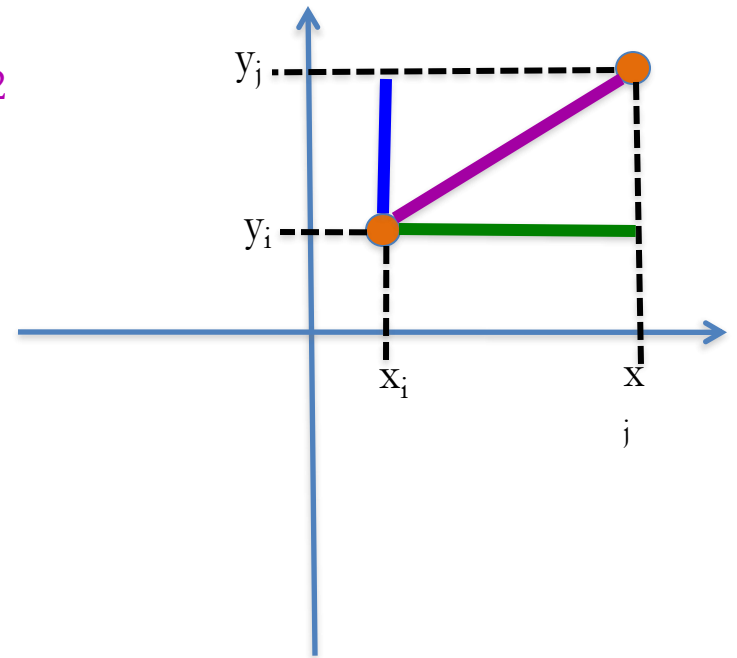
Choose the better of the two



A property of Euclidean distance

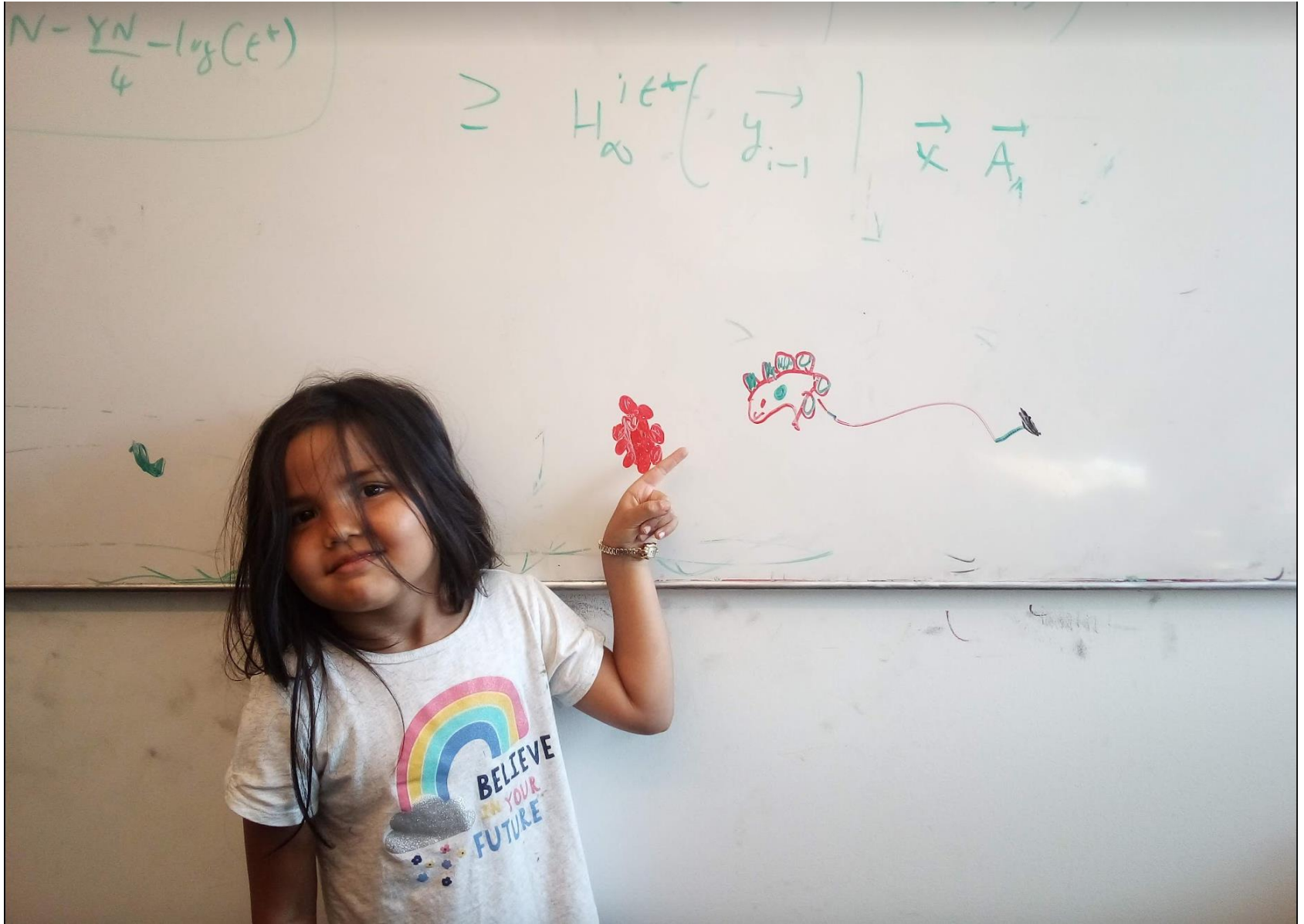


$$d(p_i, p_j) = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}$$



The **distance** is larger than the **x** or **y**-coord difference

Questions/Comments?



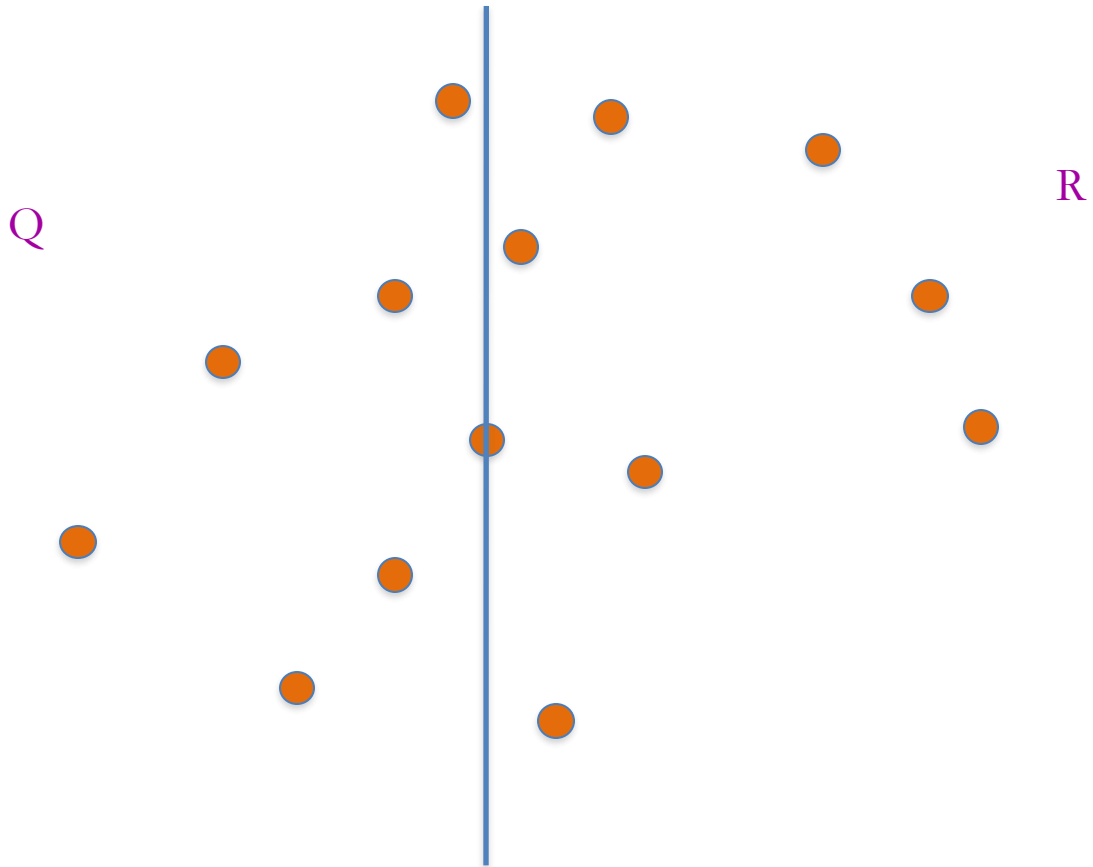
Problem definition on the board...



Rest of Today's agenda

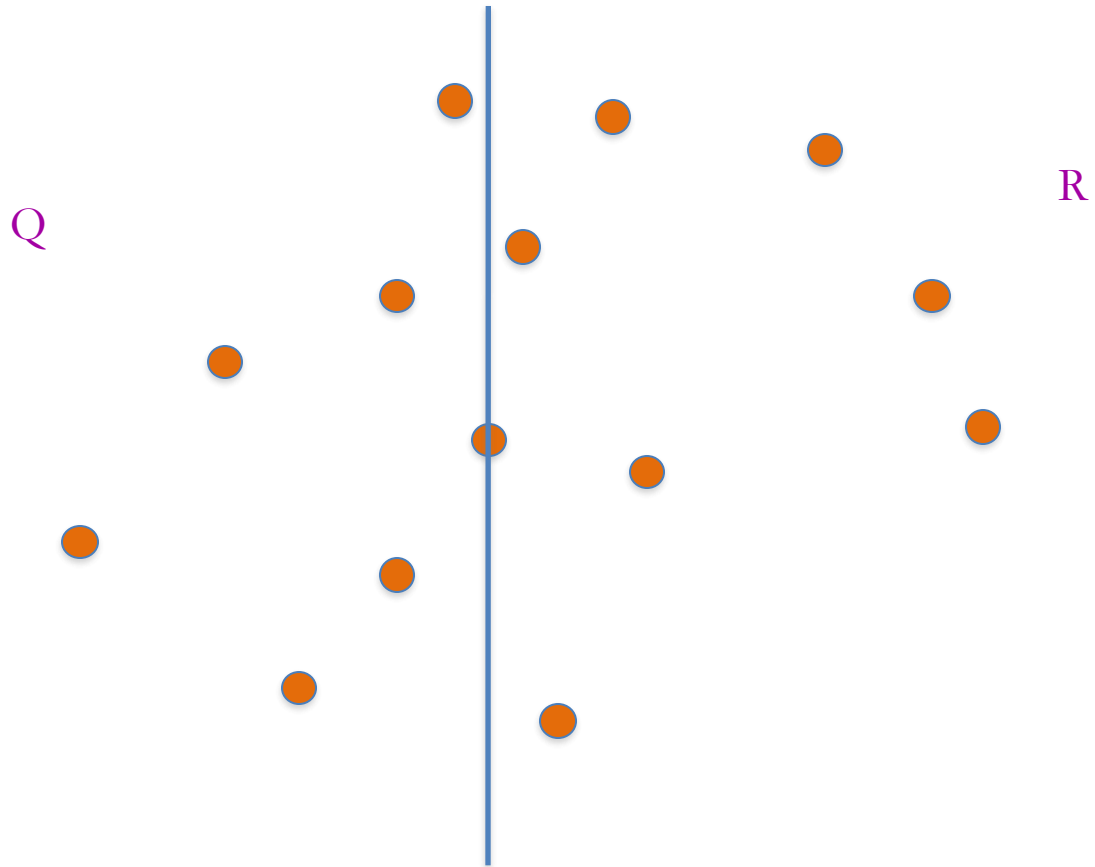
Divide and Conquer based algorithm

Dividing up P



First $n/2$ points according to the x -coord

Recursively find closest pairs



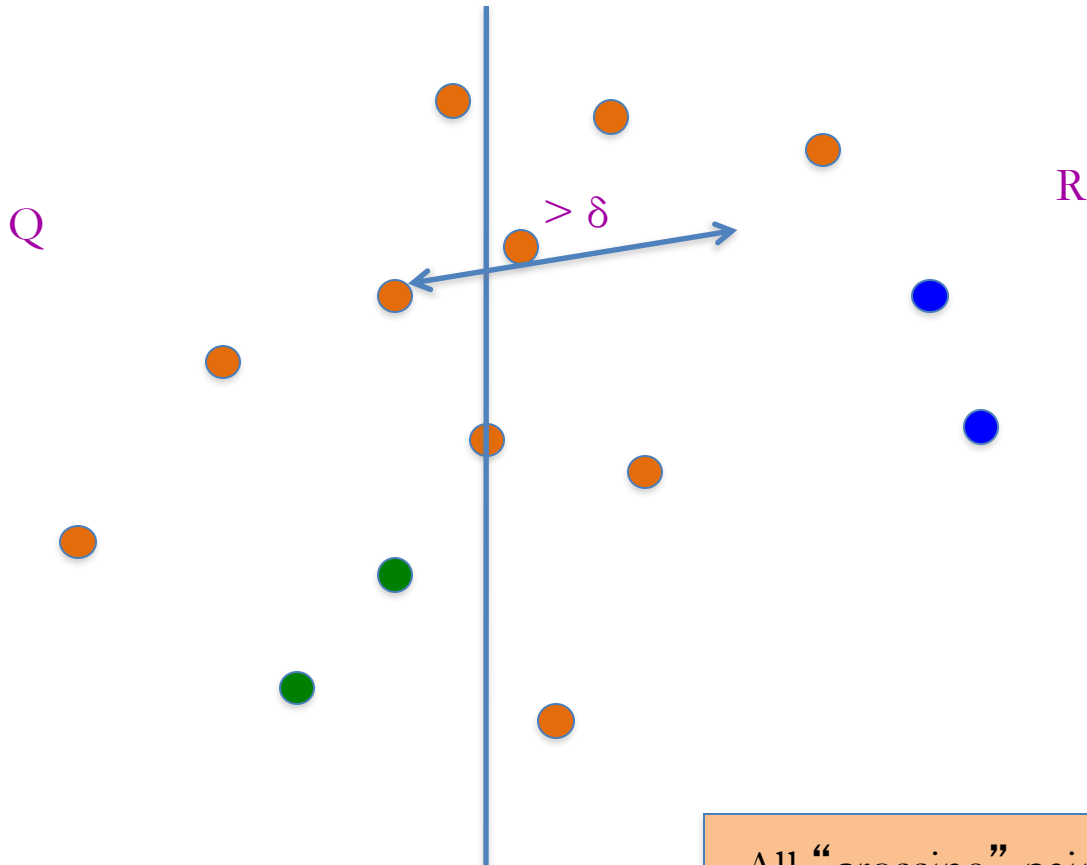
$$\delta = \min(\text{blue}, \text{green})$$

An aside: maintain sorted lists

P_x and P_y are P sorted by x -coord and y -coord

Q_x, Q_y, R_x, R_y can be computed from P_x and P_y in $O(n)$ time

An easy case

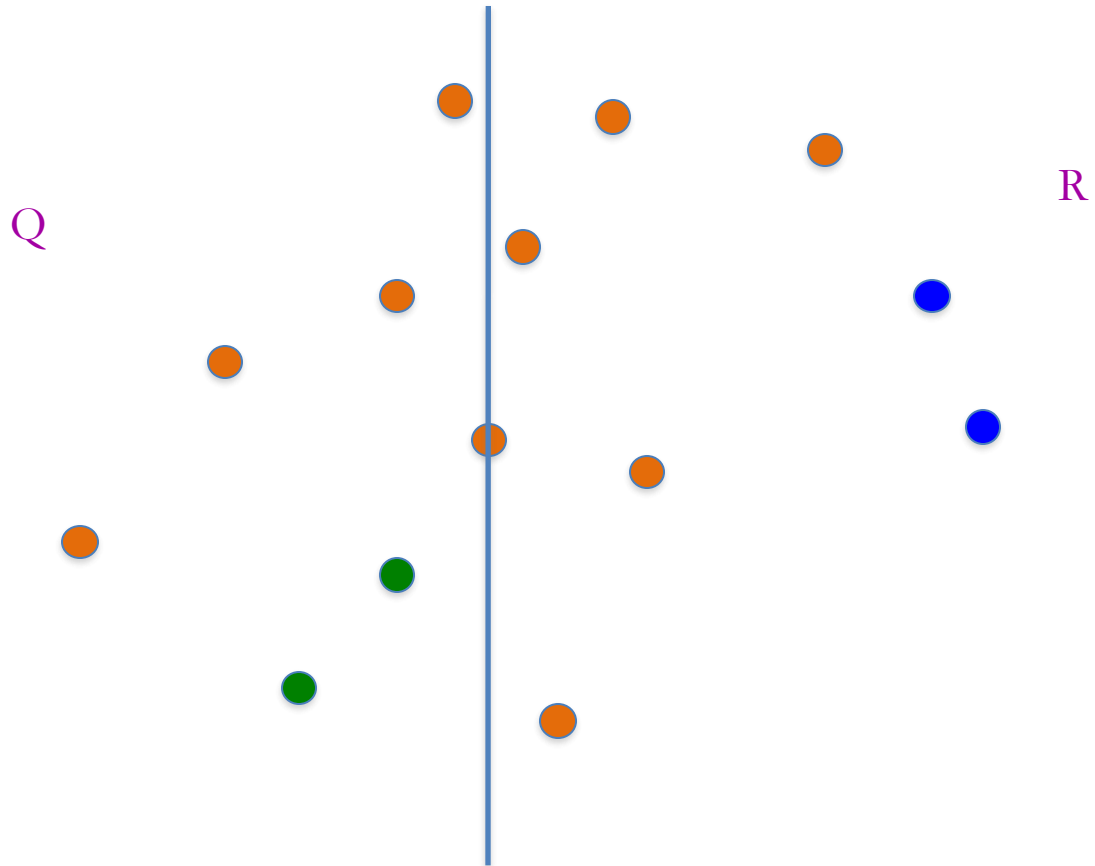


All “crossing” pairs have distance $> \delta$

$$\delta = \min(\text{blue}, \text{green})$$

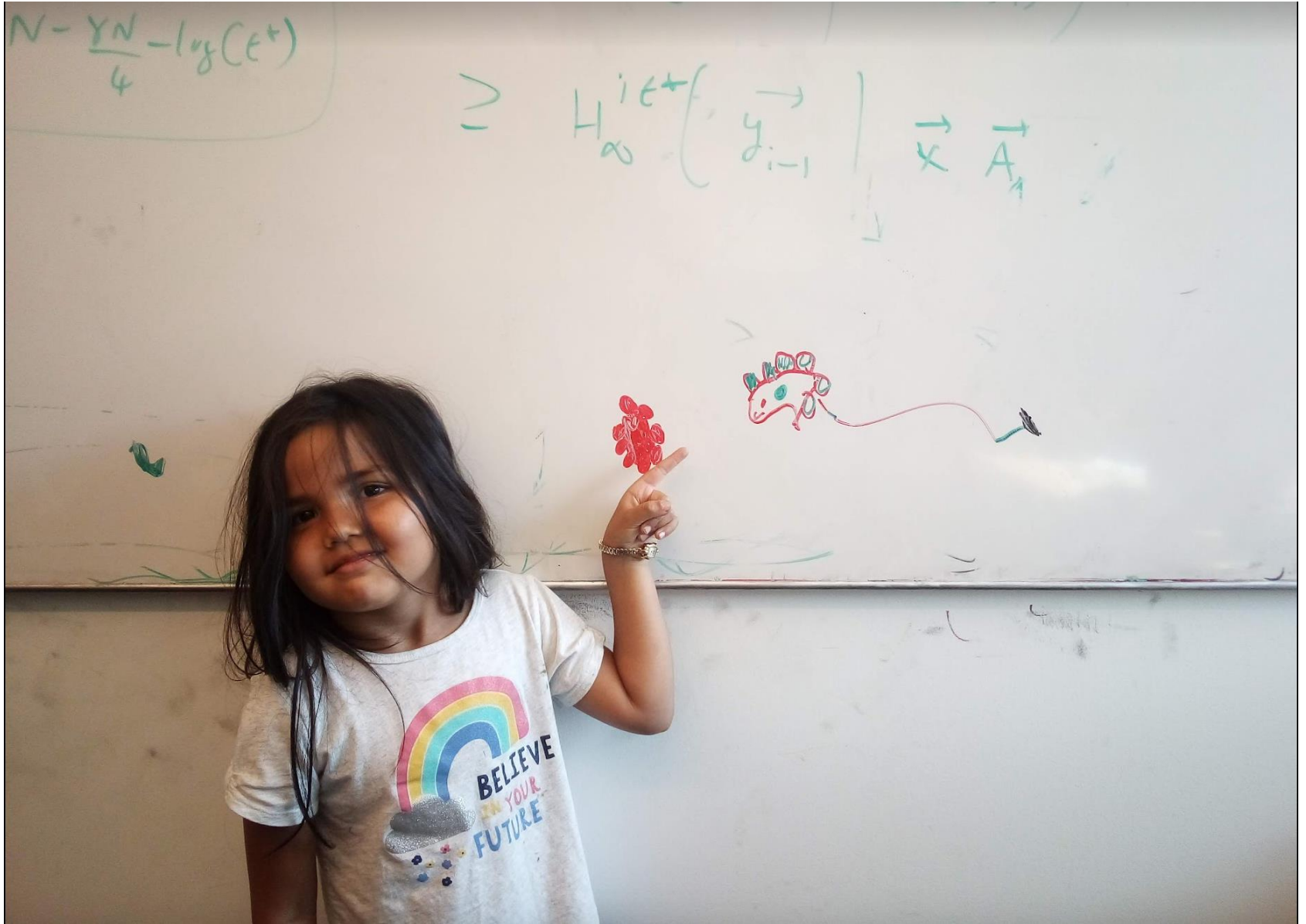


Life is not so easy though



$$\delta = \min(\text{blue}, \text{green})$$

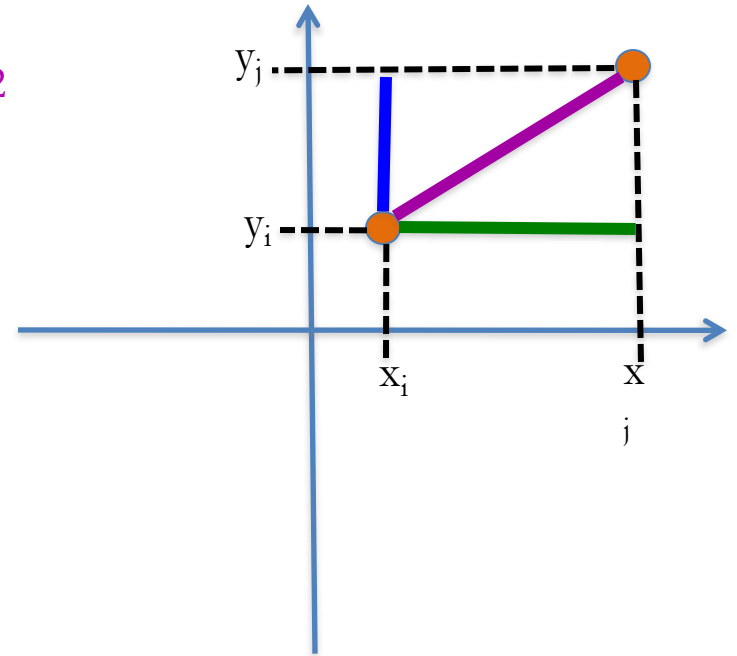
Questions/Comments?



Euclid to the rescue (?)

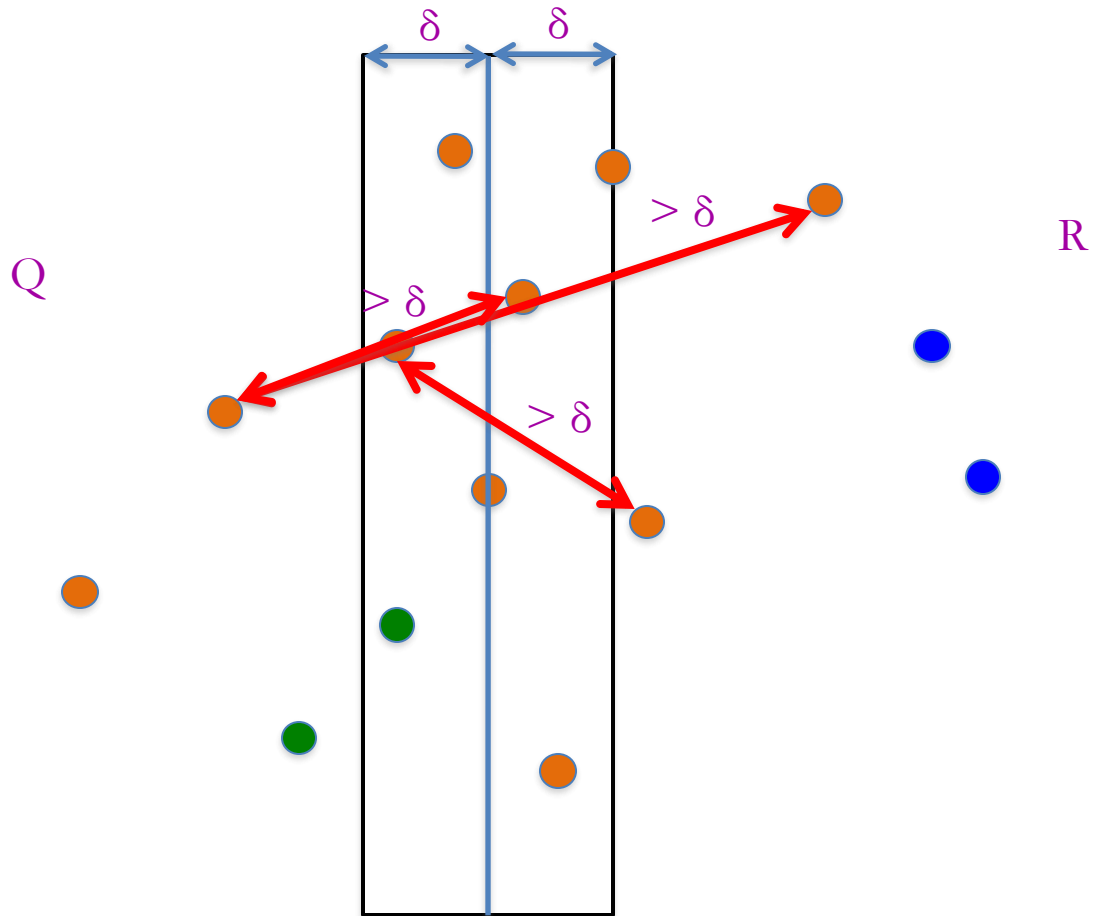


$$d(p_i, p_j) = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}$$



The **distance** is larger than the **x** or **y**-coord difference

Life is not so easy though



$$\delta = \min(\text{blue}, \text{green})$$

All we have to do now

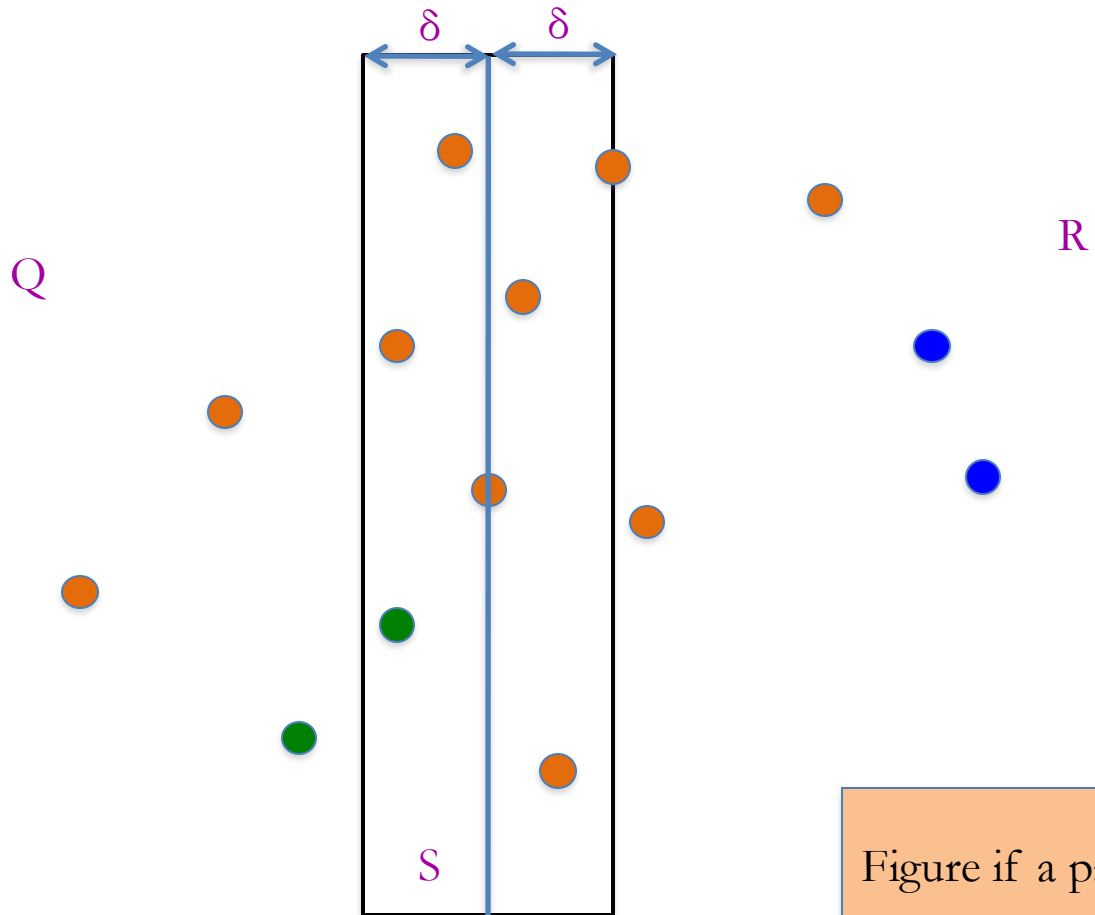


Figure if a pair in S has distance $< \delta$

$$\delta = \min(\text{blue}, \text{green})$$

The algorithm so far...

Input: n 2-D points $P = \{p_1, \dots, p_n\}$; $p_i = (x_i, y_i)$

$O(n \log n) + T(n)$

Sort P to get P_x and P_y

Closest-Pair (P_x, P_y)

$O(n \log n)$

$T(< 4) = c$

If $n < 4$ then find closest point by brute-force

Q is first half of P_x and R is the rest

$O(n)$

$T(n) = 2T(n/2) + cn$

Compute Q_x, Q_y, R_x and R_y

$O(n)$

$(q_0, q_1) = \text{Closest-Pair}(Q_x, Q_y)$

$(r_0, r_1) = \text{Closest-Pair}(R_x, R_y)$

$\delta = \min(d(q_0, q_1), d(r_0, r_1))$

$O(n)$

$S = \text{points } (x, y) \text{ in } P \text{ s.t. } |x - x^*| < \delta$

$O(n)$

return **Closest-in-box** ($S, (q_0, q_1), (r_0, r_1)$)

Assume can be done in $O(n)$

$O(n \log n)$ overall