Sep 20

A graph $G = (V, E)$ $\quad E \subseteq V \times V$

V: set of vertices/nodes $\quad$ E: set of edges

Default

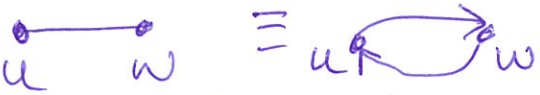
$n = |V|$

$m = |E|$

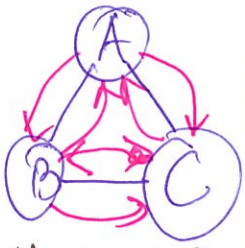
Def: G is undirected

$\Leftrightarrow \forall\ u \neq w \in V$

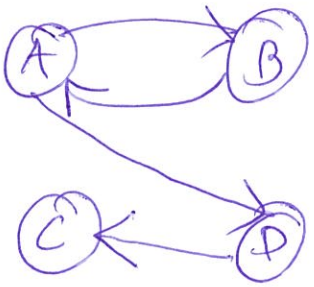
$(u,w) \in E \Leftrightarrow (w,u) \in E$



If the above is not true for all edges, then G is directed



(Undirected)



Directed

$V = \{A, B, C\}$

$E = \{(A,B), (B,C), (C,A), (A,C), (C,B), (B,A)\}$

$n = 3$

$m = \cancel{6}\ 3$

for undirected graph we explicitly keep only one of (u,w) & (w,u) $\{u,w\}$

$V = \{A, B, C, D\}$

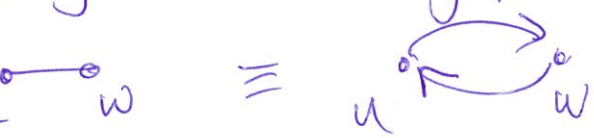
$E = \{(A,B), (B,A), (A,D), (B,C)\}$

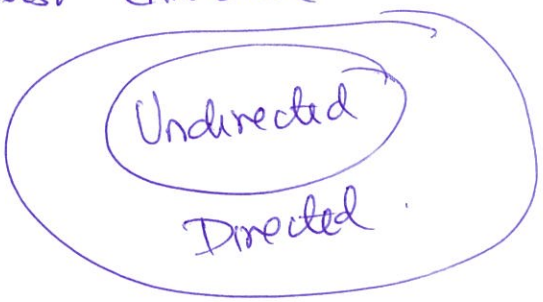
Q: Airline route map? U

Q: Wikipedia articles? D

Default: G is undirected (unless specified o/w)

Claim: Every undirected graph is also directed

Pf: 



Paths

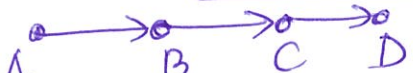
D, C, B, A ✓

✓SP → A, B, C, D ✓

×SP → A, B, C, B ✓

A, C, D ✗

A ✓



D, C, B, A ✗

A, B, C, D ✓

A, B, C, B ✗

A, C, D ✗

A ✓

Def: A path in a (directed) graph $G = (V, E)$ is a sequence of vertices $u_1, \ldots, u_k$ s.t. $\forall\ i \in \{1, \ldots, k-1\}$ $(u_i, u_{i+1}) \in E$

$\{1, \ldots, k-1\} = [k-1]$

Note: (i) u_i's need not be distinct
(ii) Also holds for directed graph

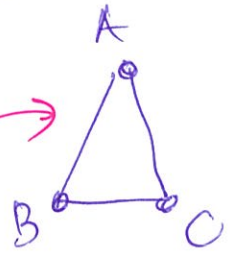
Def: A simple path is a path w/ no repeated vertices

Ex: Any simple path has length $\leq n-1$

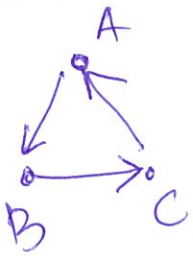
Def: Length of a path = # edges in a path.

Cycles

Triangle Graph →



A, B, C, A ✓
A, C, B, A ✓



A, B, C, A ✓
A, C, B, A ✗

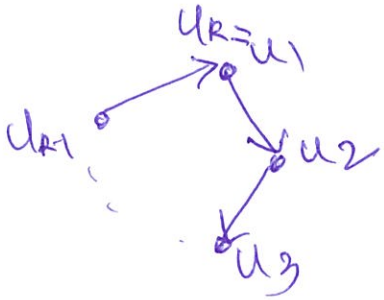
← Directed Acyclic Graph (DAG)

A, B, C, A ✗
A, C, B, A ✗

Def: A cycle in $G = (V, E)$ is a sequence $u_1, \ldots, u_{k-1}, u_k = u_1$ and $u_1, \ldots, u_{k-1}$ are distinct

s.t. $i \in [k-1]$ $(u_i, u_{i+1}) \in E$

✗



Condition on k

(i) Directed graph $k \geq 3$
(ii) Undirected graphs $k \geq 4$

A, B, A ✗