Sep 23 **PROPOSITION** Let T be a BFS tree for $G=(V,E)$

If $(u,w) \in E$ s.t. $u \in L_i$, $w \in L_j$

$$\Rightarrow |i-j| \le 1 \iff j \in \{i-1, i, i+1\}$$

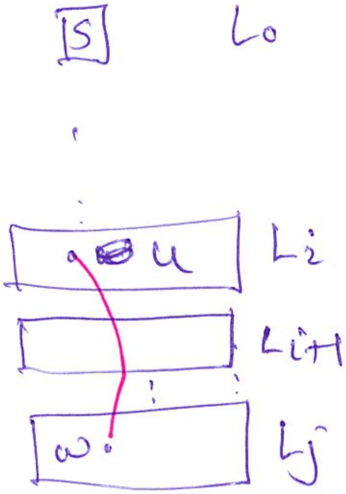
Pf (idea): By contradiction.

WLOG assume $i \le j$ {o/w switch the roles of i & j in the pf below}

Without loss of generality

For the sake of contradiction $|i-j| > 1$

$$\Rightarrow j > i+1 \iff j \ge i+2$$



Consider the situation when BFS is about to create L_{i+1}

(•) $u \in L_i$, $w \notin L_0, \dots, L_i$

(•) $(u,w) \in E$

$\Rightarrow$ (by BFS algo def) w will be added to L_{i+1}

$\Rightarrow$ contradicts $w \in L_j$ for $j \ge i+2$ ∎

Explore (s, G)

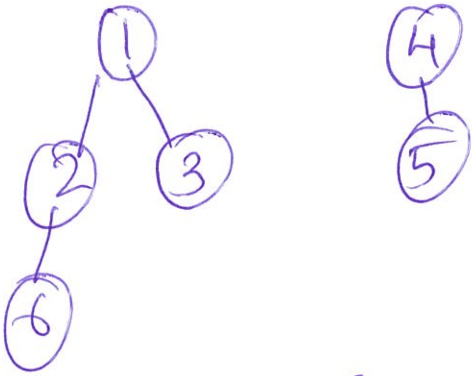
0. $R \leftarrow \{s\}$
1. While $\exists\ (u,w) \in E$ s.t. $u \in R$, $w \notin R$

   Add w to R

2. Output $R^* \leftarrow R$

Lemma: Explore always terminates (Ex $\rightarrow$ later on we'll show BFS runs in $O(n+m)$ time)

Def: Set of all vertices connected to s in G is the connected component of s $CC(s)$



$CC(2) = \{1, 2, 3, 6\}$

$CC(5) = \{4, 5\}$

THM: For all graphs $G = (V, E)$, $s \in V$

$CC(s) = R^* \leftarrow$ (when call Explore (s, G))

BFS is a special case of Explore $\Longrightarrow$ BFS is correct

General trick: To show 2 set $A = B$

show (1) $A \subseteq B$ and (2) $B \subseteq A$

Lemma 1: $R^* \subseteq CC(s) \leftarrow$ everything output by Explore is correct

EX (induction)

Lemma 2: $CC(s) \subseteq R^* \leftarrow$ everything in $CC(s)$ is output by Explore

$\uparrow$ next lecture.