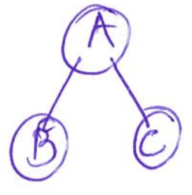
$\boxed{\text{Sep 27}}$

Lemma: For every undirected graph

$$2m = \sum_{u \in V} n_u$$

(Undirected) graph $G = (V, E)$

degree of u → n_u = # neighbors of u $= \left| \{ w \mid (u,w) \in E \} \right|$



$n = 3$

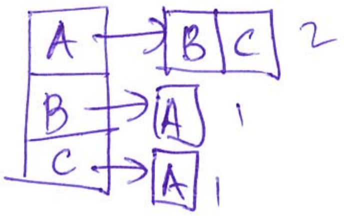
$m = 2$

$n_A = 2$

$n_B = 1$

$n_C = 1$

$$\underset{\underset{2m}{\parallel}}{2 \cdot 2} = \underset{\underset{n_A}{\uparrow}}{2} + \underset{\underset{n_B}{\uparrow}}{\overset{1}{\cancel{n_B}}} + \underset{\underset{n_C}{\uparrow}}{\overset{1}{\cancel{n_C}}} = 4$$

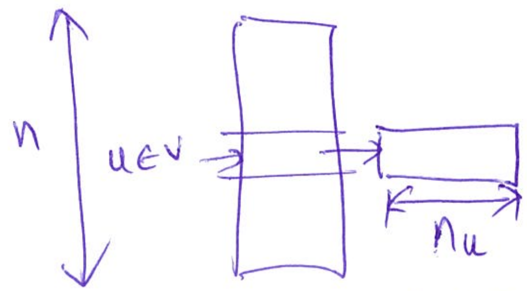


ptrs = 3

size of all lists = 4

$$\text{total sz} = \underbrace{3}_{n} + \underbrace{4}_{2m}$$

Adj. List for a general G



ptrs = n

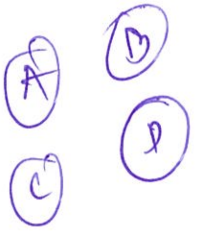
sum of list sizes

$$= \sum_{u} n_u$$

$$= 2m$$

$$\Rightarrow \text{total size} = N = n + 2m = \Theta(n+m)$$

$O(n) \leq$ $\leq O(n^2)$



$$\overset{?}{0} \leq m \leq \overset{?}{\binom{n}{2}} = \frac{n(n-1)}{2} < \frac{n^2}{2} \leq O(n^2)$$

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BFS (G, s)      // G is adj. list

O(n) → 0. CC[s] ← 1 and CC[u] = 0 $\forall u \neq s \in V$
                                  // CC is an array
O(1) { 1. i ← 0
       2. $L_0$ ← {s}     // $L_j$ is a linked list $\forall j$
       3. While $L_i \neq \phi$ ← empty set                    ≤n
              3.1 $L_{i+1}$ ← {}  // O(1)
              3.2 for u ∈ $L_i$                               ≤n
                     for (u,w) ∈ E                           ≤n
                         if CC[w] = 0                         This block is run ≤ T tim
                            CC[w] ← 1                         O(1)
                            Add w to $L_{i+1}$
  O(1)        3.3 i ← i+1
       4. Return CC  // Connected comp of s = {w | CC[w] = 1}
  O(n)
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Overall runtime $= O(n) + T \cdot O(1)$

$= O(n) + O(T)$

$= O(n+T)$

$= O(n+m)$

$T \leq n^3$

$T \leq O(m)$

See book for: {book}