Oct 4

Greedy Algo

0. $R \leftarrow [n]$
1. $S \leftarrow \phi$
2. While $R \neq \phi$ ← OK since $f(1) \leq \ldots \leq f(n)$
   (2.1) Let i be the smallest index in R
   (2.2) Add i to S
   (2.3) Delete all j (from R) that conflict with i ← i conflicts with i itself
3. Return $S^* \leftarrow S$

THM 1: S^* is an optimal solution ← for all inputs, among all possible valid schedules S for that input, S^* has the max # intervals $|S^*| \geq |S|$

Pf of correctness
- → Greedy stays ahead (next)
- → Exchange argument (min max lateness → Sec 4.2)

Greedy stays ahead

Order					
	S^*	$i_1,$ ..	$\boxed{i_r}$	i_m
	Θ	$j_1,$ -	$\boxed{j_r}$	· j_k

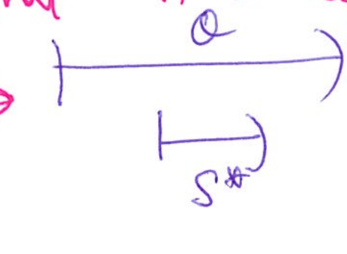
S^* is doing "just as well" the Θ

Pf (idea) of THM 1:

Let Θ be an optimal

NOTE: Cannot say anything (e.g. a structural property) about Θ other than the fact that it is optimal

~~THM 2~~: $S^* = \Theta$ X →



THM 2: $|\underset{\shortparallel\, k}{S^*}| = |\underset{m}{\Theta}|$

Notation: $S^* = \{i_1, \ldots, i_k\}$ $\qquad f(i_1) \le f(i_2) \le \cdots f(i_k)$

$\mathcal{O} = \{j_1, \ldots, j_m\}$ $\qquad f(j_1) \le f(j_2) \le \cdots \le f(j_m)$

THM 3: $k = m$

EX

Claim 1: $k \le m$ (since $\mathcal{O}$ is opt)

THM 4: $k \ge m$ (THM 4 + Claim 1 $\Rightarrow$ THM 3)

Lemma 1 (Greedy stays ahead) $\forall \; 1 \le \ell \le \boxed{k}$

$$f(i_\ell) \le f(j_\ell)$$

(Assume Lem 1)

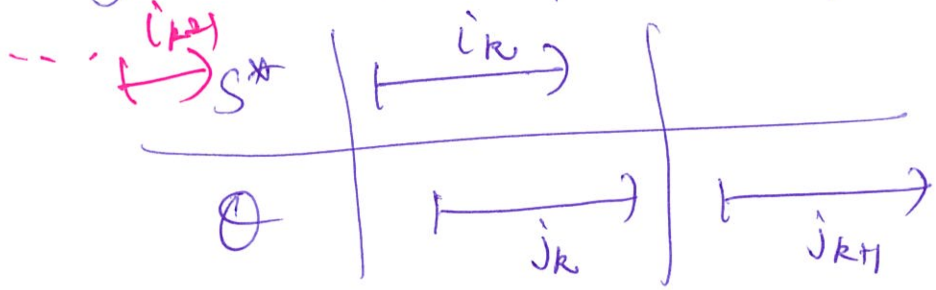
Pf (idea) of THM 4: Pf by contradiction

Recall: Greedy algo has terminated (*)

Assume for sake of contradiction $k < m$

or $m \ge k+1 \Rightarrow j_{k+1} \in \mathcal{O}$

By ~~Lem~~ Lem 1 $\quad f(i_k) \le f(j_k)$



Consider the situation right after i_k is added to S

Claim (EX): $j_{k+1} \in R$

j_{k+1} doesn't conflict w/ $i_k, i_{k-1}, \ldots$

$\underset{\text{algo def}}{\Longrightarrow}$ in the next iteration $R \ne \emptyset \Longrightarrow$ Greedy cannot have terminated

$\blacksquare$ contradicts (*) $\Longleftarrow$

Pf (idea) of Lem 1: Pf by induction on ℓ

Base case: $\ell = 1$

$$f(i_1) \leq f(j_1)$$

$i_1 = 1 \quad \Uparrow$

$f(1) \leq f(2) \leq \ldots$ ✓

IH: for some r s.t. $1 \leq r < k$

$\forall\ 1 \leq \ell \leq \boxed{r} \quad f(i_\ell) \leq f(j_\ell)$

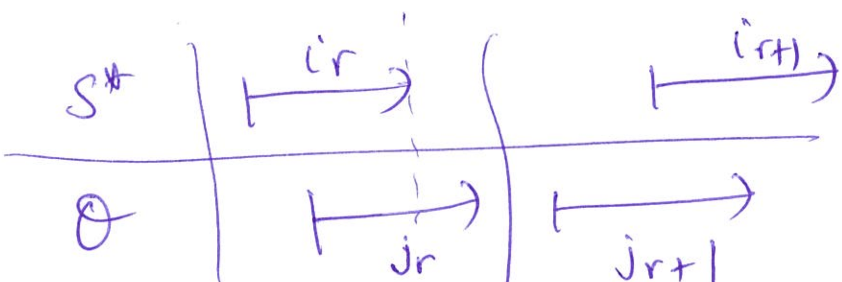
I.S: $f(i_{r+1}) \leq f(j_{r+1})$ $\quad r+1 \leq k$

By contradiction.

Note: $i_{r+1} \in S^*$ ——— (#)

$f(i_{r+1}) > f(j_{r+1})$ $\quad$ otoh $\quad$ By IH $f(i_r) \leq f(j_r)$

On the other hand



Consider the situation after i_r is added to S

$\Rightarrow i_{r+1}, j_{r+1} \in R$

But since we have $f(i_{r+1}) > f(j_{r+1})$ Greedy cannot pick i_{r+1} ~~for~~ j_{r+1}.

$\Rightarrow$ contradicts (#) ∎