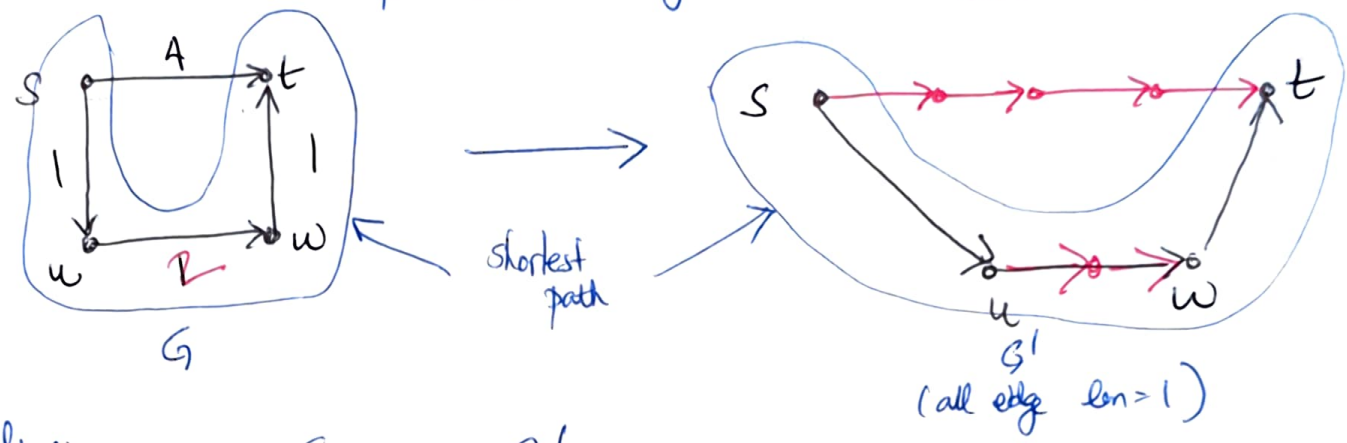
Oct 16

Reduction from Friday

→ Replace each edge e (with integer edge length $l_e \geq 0$) with a path of length l_e ← new paths do not share edges



Redux:

$G \longrightarrow G'$

$\{l_e\} \qquad \{l'_e = 1\}$

→ Run the algo from HW 3 Q3 on G'.

→ If P' is the shortest s-t path in G'

↳ return the corresponding path P in G

Ex: This is a correct algo.

$G = (V, E)$, $G' = (V', E')$

runtime of HW 3 Q3 algo = $O(|V'| + |E'|)$

$n' \stackrel{\text{def}}{=} |V'|$, $m' \stackrel{\text{def}}{=} |E'|$

Ex: Reduction runtime: constructing G' from G & constructing P from P' $\quad O(n' + m')$

OVERALL: $O(n' + m')$ Input size = $\Theta(n+m)$

Q: How does $n' + m'$ relate to $n + m$?

$n' \leq l_{max}\, n$

$m' \leq l_{max}\, m$

$l_{max} = \max_{e \in E} l_e$

→ $O(l_{max}(n+m))$ runtime

$\ell_{max} = O(1) \Rightarrow O(n+m)$ ✓

$\ell_{max} = n^{100} \rightarrow O(n^{100}(n+m))$

ASIDE: RAM model unit of space is a register

If your input has n items, each register is $O(\log n)$ bits long.

$\Rightarrow$ Adj list has $O(n+m)$ registers.

Q: How many registers would you need to holds integers $1 .. n^{100}$?

Q': How many bits will you need

$\sim \log n^{100}$ bits $= 100 \log n \approx$ 100 registers

$O(1)$ registers

$\ell_{max} \leq n^{O(1)} \Rightarrow O(1)$ registers.

assume this.