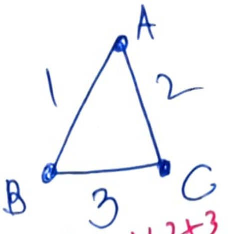
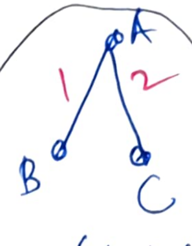
Oct 21

$G = (V, E)$

connected → G; undirected → E

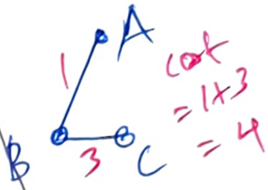


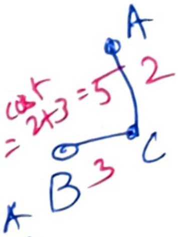
cost = 1+2+3 = 6

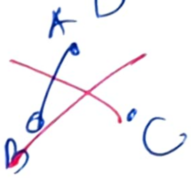


$T = (\{A, B, C\}, \{(A,B), (A,C)\})$

cost = 1+2 = 3







$\forall e \in E, \quad c_e \geq 0$

(all algos we'll use can handle -ve $c_e < 0$)

Output: $E' \subseteq E$ s.t

(i) $T = (V, E')$ to be connected — spanning subgraph

(edge) subgraph of G

(ii) $c(T) = \sum_{e \in E'} c_e$ is min

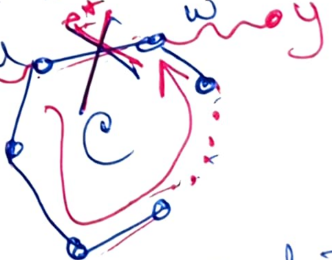
Minimum Spanning Tree (MST)

Prop: Let $c_e > 0 \ \forall e \in E$, then every optimal solution $T = (V, E')$ is a tree. (*)

Pf (details) By contradiction. Let $T = (V, E')$ be an optimal solution that is not a tree.

$\Rightarrow$ T has a cycle C

T is connected



Goal: Show another spanning subgraph $T' = (V, \bar{E})$ $\bar{E} \subseteq E$ & T' is connected

and $c(T') < c(T)$

If such as T' exists $\Rightarrow$ contradicts (*)

→ Pick any edge $e^* = (u, w)$ from C & drop it

$\bar{E} = E' \setminus \{e^*\}$ set difference

Claim 1: $c(T') < c(T)$

Pf: $c(T') = \sum_{e \in \bar{E}} c_e = \sum_{e \in E'} c_e - c_{e^*}$

$= c(T) - c_{e^*}$ ⇐ as $c_{e^*} > 0$

$\leq c(T)$

Claim 2: T' is connected.

Pf: Fix any $x, y \in V$

Case 1: $\exists$ an x-y path not using e^*. ⇒ x, y still connected in T'

Case 2: x-y path uses e^*

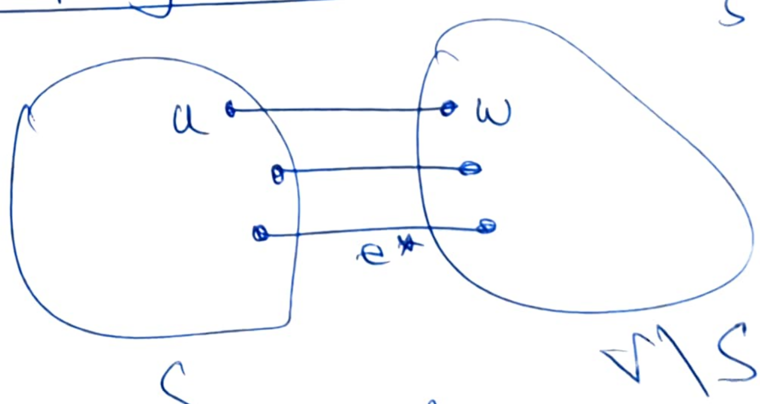
If x-y path uses e, use the "second route" in $C \setminus \{e^*\}$
⇒ x, y still connected in T' ■

Assume: All c_e are distinct

CUT Property Lemma

$S \neq \emptyset$

$S \neq V$



Let e^* be the cheapest edge

⇒ e^* is in ALL MSTs.