

Oct 25

$T(n) \rightarrow$ Merge Sort (a, n)

floor $\lfloor 0.3 \rfloor = 0$
ceil $\lceil 0.3 \rceil = 1$

$O(1)$ $\left\{ \begin{array}{l} \text{if } n=1: \\ \text{return } a_1 \end{array} \right.$

$O(n)$ $\left\{ \begin{array}{l} a_L \leftarrow a_1, \dots, a_{\lfloor \frac{n}{2} \rfloor} \\ a_R \leftarrow a_{\lfloor \frac{n}{2} \rfloor + 1}, \dots, a_n \end{array} \right. \leq T(\lfloor \frac{n}{2} \rfloor)$

return MERGE(MergeSort $(a_L, \lfloor \frac{n}{2} \rfloor)$,
MergeSort $(a_R, n - \lfloor \frac{n}{2} \rfloor)$)
 \uparrow $\leq T(n - \lfloor \frac{n}{2} \rfloor)$
 \uparrow $O(n)$

$T(n) \stackrel{\text{def}}{=} \max_{\forall n}$ runtime of MergeSort over all inputs of size n

THM: $T(n)$ is $O(n \log n)$

$n=1$, $T(1) \leq O(1)$

$n > 1$:
$$T(n) \leq O(1) + O(n) + T(\lfloor \frac{n}{2} \rfloor) +$$

$$\lceil \frac{n}{2} \rceil T(\lceil \frac{n}{2} \rceil) + O(n)$$

$$\leq O(n) + T(\lfloor \frac{n}{2} \rfloor) + T(\lceil \frac{n}{2} \rceil)$$

$$\approx O(n) + T(\frac{n}{2}) + T(\frac{n}{2})$$

$$= O(n) + 2T(\frac{n}{2})$$

$T(n) \leq \begin{cases} O(1) & \text{if } n=1 \\ O(n) + 2T(\frac{n}{2}) & \text{o/w} \end{cases}$