Nov 4

Closest pair of points $x_i, y_i \le poly(n)$

Input: n distinct points $p_1, \dots, p_n$ $p_i = (x_i, y_i)$ integers

Output: pair p_i, p_j s.t. $d(p_i, p_j)$ is min

$$d(p_i, p_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Assumptions:

(1) Given p_i & p_j we can compute $d(p_i, p_j)$ in $O(1)$ time.

$d(p_i, p_j)$ is min $\iff$ $d(p_i, p_j)^2$ is min.

(2) Assume all x_i values are ~~dist~~ distinct

—— y_i ——

If not, (i) (Claim) $\exists$ a rotation of n distinct pts s.t. after rotation the assumption holds.

(ii) Can modify the algo we'll see later to handle the general case

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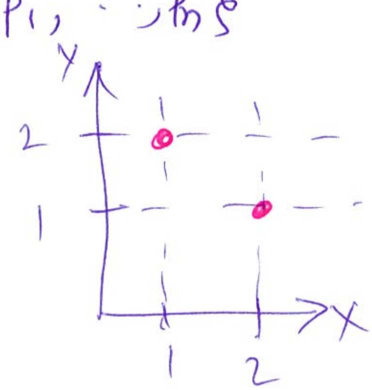
Notation: P is the set of all points $P = \{p_1, \dots, p_n\}$

Eg. $P = \{(1,2), (2,1)\}$



P_X is P sorted by x-values

Eg $P_X = \{(1,2), (2,1)\}$

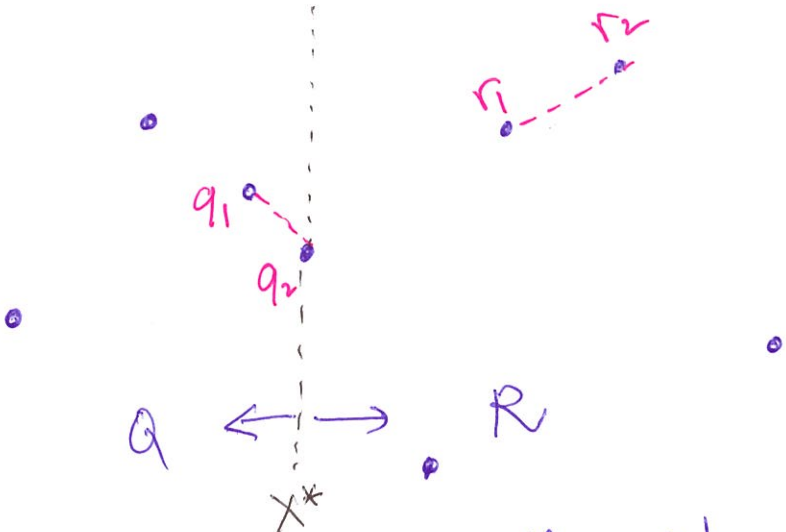
P_Y is P sorted by y-values

Eg $P_Y = \{(2,1), (1,2)\}$

Goal: Do better than $O(n^2)$ algo

Towards a Divide & Conquer algo

n=8



Indexings starts at 1

<u>Step 1</u>: Divide P equally into Q & R

$$(x^*, y^*) = P_x\left[\left\lceil \frac{n}{2} \right\rceil\right]$$

$$Q = \{(x,y) \in P \mid x \leq x^*\}$$

$$R = \{(x,y) \in P \mid x > x^*\}$$

$$|Q| = \left\lceil \frac{n}{2} \right\rceil \qquad |R| = n - \left\lceil \frac{n}{2} \right\rceil = \left\lfloor \frac{n}{2} \right\rfloor$$

<u>Step 2</u>: Recursively find closest pair of points in Q and R

$q_1, q_2 \in Q$ are closest pts in Q

$r_1, r_2 \in R$ ________ R

<u>IMPORTANT</u> : $\delta = \min\left(d(q_1, q_2), d(r_1, r_2)\right)$

<u>ASIDE</u>: P_X, P_Y from P in $O(n \log n)$ time

<u>Q</u>: Given P_X, P_Y: compute Q_X, Q_Y, R_X, R_Y is $O(n)$ time.

A: $Q_X = P_X\left[1 : \left\lceil \frac{n}{2} \right\rceil\right]$, $R_X = P_X\left[\left\lceil \frac{n}{2} \right\rceil + 1 : n\right]$

$\forall (x,y) \in P_Y$ if $x \leq x^*$ add (x,y) to Q_Y
else add (x,y) to R_Y

$O(n)$ time.