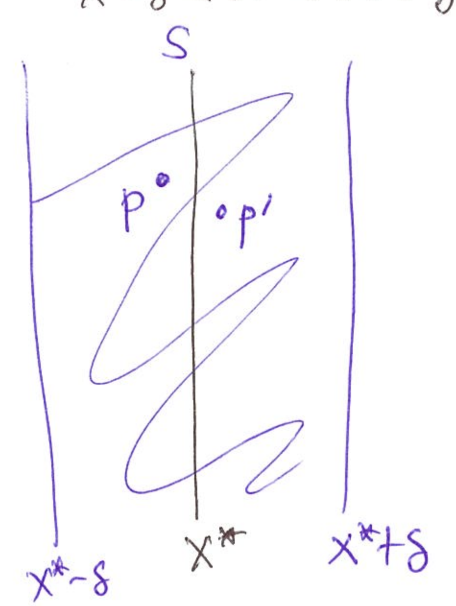
Nov 6

Closest-in-box (S) ← returns closest pair $p \neq p' \in S$ IF ~~d(...)~~ $d(p,p') < \delta$ else NULL

$S = \{(x,y) \in P \mid |x^* - x| < \delta\}$ ↔ $x^* - \delta < x < x^* + \delta$



KICKASS PROPERTY LEMMA

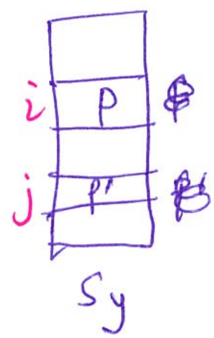
For every $p \neq p' \in S$ s.t. $d(p,p') < \delta$

if $p = S_y[i]$

$p' = S_y[j]$

then $|i - j| \leq 15$ (!)

Note: Can replace "15" by 9 (or even 7)



Q: $O(n)$ time algo for Closest-in-box?

$n' = |S|$ $\qquad n' \leq n$

for $i = 1 \ldots, n'-1$

check $(S_y[i], S_y[i+1])$, $(S_y[i], S_y[i+2])$ ← min$(i+2, n')$

$\ldots$ $(S_y[i], S_y[i+15])$ ← min$(i+15, n')$

Let (p_i, p_i') be the closest pair of pts among

$O(1)$ $\qquad O(n')$

Let (p, p') be the closest pair among (p_i, p_i') $i = 1 \ldots n'-1$ ← $O(n')$

If $d(p,p') < \delta$

return (p, p')

else NULL

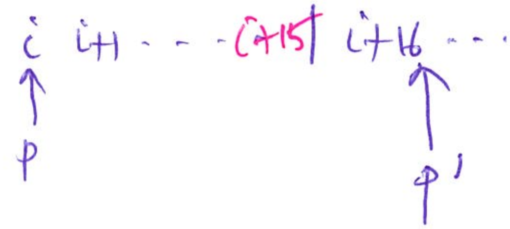
$O(1)$

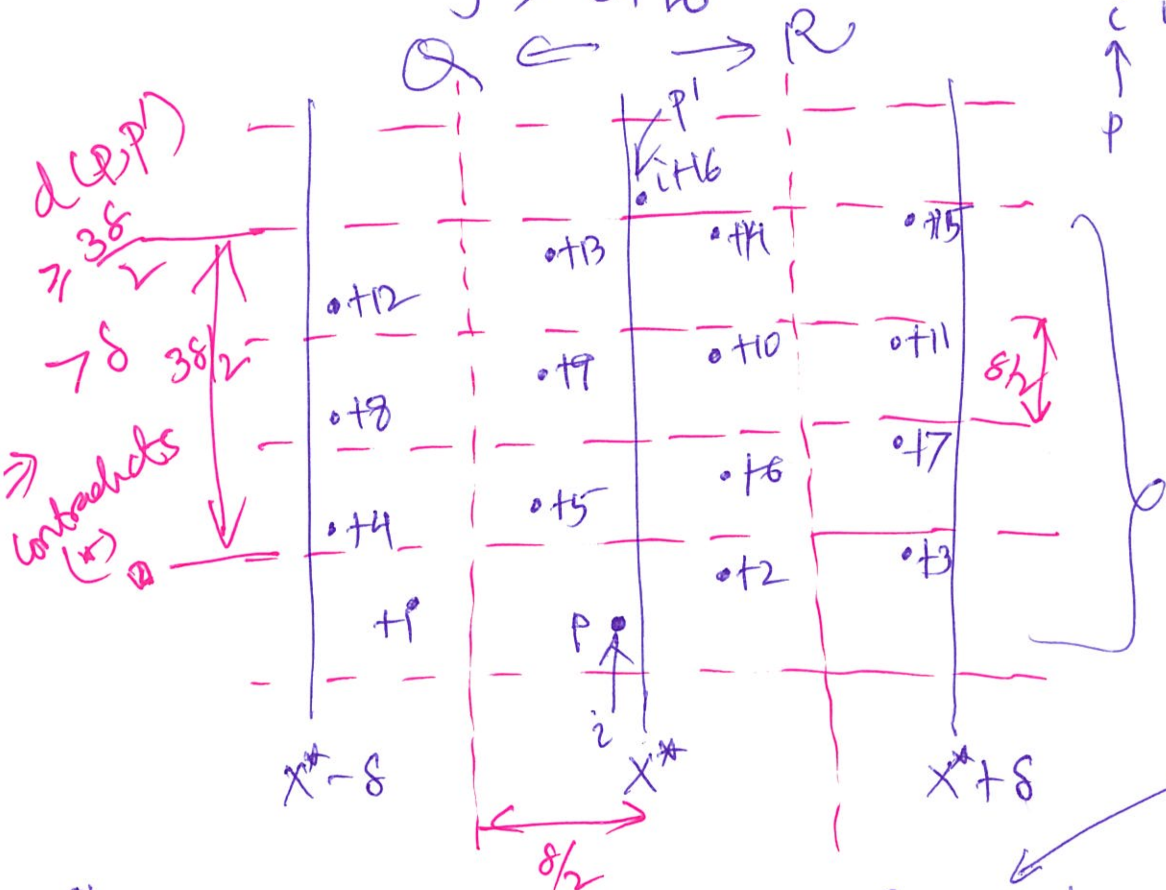
OVERALL: $O(n') \leq O(n)$

Pf (idea) of Kickass Property Lemma

(*) By contradiction. Assume $\exists$ $p, p' \in S$ s.t. $d(p,p') < \delta$ with $p = S_y[i]$, $p' = S_y[j]$ but { Assume $j \geq i$ }

$j \geq i + 16$





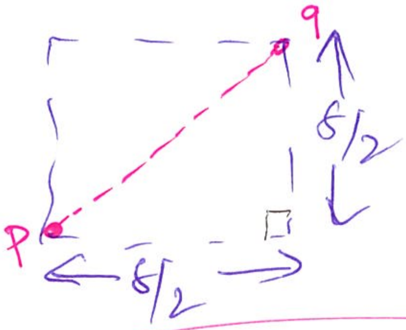
i+16 cannot go in there

note: each box completely in Q or in R

Claim: Every $\frac{\delta}{2} \times \frac{\delta}{2}$ square boxes have ≤ 1 pt in this.

Pf (idea) By contradiction. Assume $\exists$ pts p & q inside of one box

Ef) p & q are furthest apart if they are on the diagonal



$$d(p,q) = \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2} = \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta$$

as $\sqrt{2} > 1$

contradicts the fact / defn of δ
(note: p & q are either both in Q or ——— in R