Nov 13

Iterative-Compute-Opt // $M[0..n]$

① $M[0] \leftarrow 0$

② for $j = 1..n$

$M[j] \leftarrow \max\{v_j + M[p(j)], M[j-1]\}$

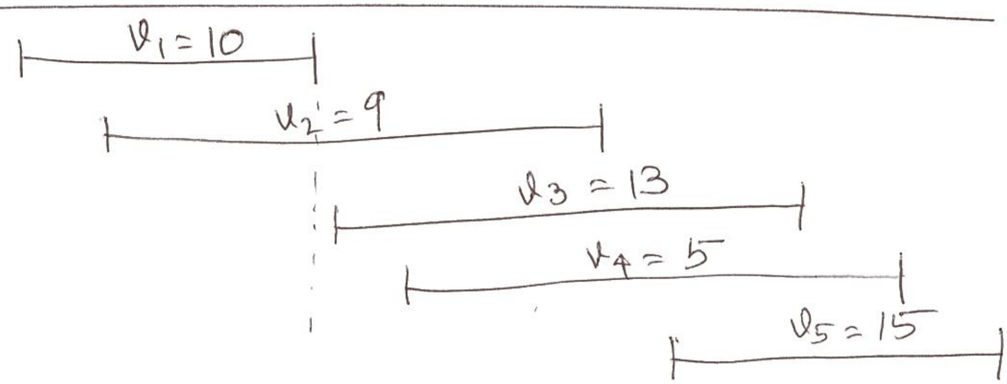
③ return ~~$M[n]$~~ $MSchedule(n; M, p)$

Assume
(i) $f_1 \le f_2 \le \dots \le f_n$
(ii) have access to $p(1), \dots, p(n)$

Can ensure (i)+(ii) in $O(n \log n)$ time

$n=5$

$p(1) = 0$
$p(2) = 0$
$p(3) = 1$
$p(4) = 1$
$p(5) = 2$



M

	0	1	2	3	4	5
$j=0$	0					
$j=1$	0	10				
$j=2$	0	10	10			
$j=3$	0	10	10	23		
$j=4$	0	10	10	23	23	
$j=5$	0	10	10	23	23	25

$M[0] \leftarrow 0$

$M[1] = \max\{v_1 + M[0], M[0]\}$
$= \max\{10 + 0; 0\} = 10$

$M[2] = \max\{v_2 + M[0], M[1]\}$
$= \max\{9 + 0, 10\} = 10$

$M[3] = \max\{v_3 + M[1], M[2]\}$
$= \max\{13 + 10, 10\} = 23$

$M[5] = \max\{v_5 + M[2], M[4]\}$
$= \max\{15 + 10, 23\} = 25$

return as $OPT(n)$

$O_5 = \{1, 5\}$

Recall: $j \in O_j \iff v_j + OPT(p(j)) > OPT(j-1)$

$\boxed{n=5}$ $\qquad \underset{M[p(j)]}{\overset{\shortparallel}{}} \qquad \underset{M[j-1]}{\overset{\shortparallel}{}}$ Also do $\geq$

$5 \overset{?}{\in} O_5$

$\iff v_5 + M[p(5)] \overset{?}{>} M[4]$

$15 + 10 \overset{?}{>} 23$ ✓ $\Rightarrow 5 \in O_5$

$p(5) = 2$, $O_5 \setminus \{5\}$ is optimal $[p(j)]$
$= O_2 \qquad\qquad = [2] = \{1, 2\}$

$\iff O_5 = \{5\} \cup O_2$

$\rightarrow 2 \overset{?}{\in} O_2 \iff v_2 + M[0] \overset{?}{>} M[1]$

$\times \quad 9 + 0 \overset{?}{>} 10$

$2 \notin O_2 \Leftarrow$

$\Rightarrow O_2$ is optimal for $[1]$
$= O_1$

$1 \overset{?}{\in} O_1 \iff v_1 + M[0] \overset{?}{>} M[0]$

$\iff 10 + 0 \overset{?}{>} 0$ ✓ $\Rightarrow 1 \in O_1$

$\Rightarrow O_1 = \{1\} \Rightarrow O_2 = \{1\} \Rightarrow O_5 = \{5\} \cup \{1\} = \{1, 5\}$

MSchedule $(n; M, p)$

$O(n)$ time

```
If n = 0, return ϕ
If v_n + M[p(n)] > M[n-1]
    return {n} ∪ MSchedule (p(n); M, p)
else
    return MSchedule (n-1; M, p)
```