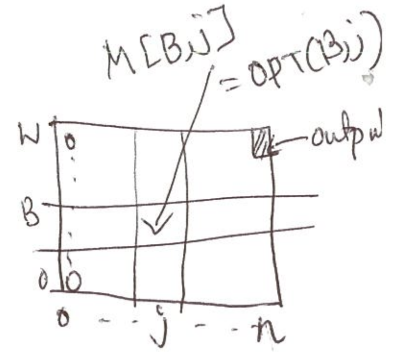
Nov 10

$\swarrow O(nW)$ SubsetSum $(w_1, \ldots, w_n; W)$



0. Allocate $(W+1) \times (n+1)$ matrix M

$\rightarrow$ 1. $M[B,0] \leftarrow 0 \quad \forall\ 0 \le B \le W$

$O(W)$ 2. for $j = 1 .. n$

$O(nW)$ for $B = 0 .. W$

$O(1)$ if $w_j > B$ then $M[B,j] \leftarrow M[B,j-1]$

else

$M[B,j] \leftarrow \max\{ M[B,j-1],\ w_j + M[B-w_j, j-1]\}$

3. Return $M[W,n] \leftarrow O(1)$

Prove correctness by induction (Ex.)

OVERALL $= O(nW) + O(W) + O(nW) + O(1)$

$= O(nW) \leftarrow$ If $W \le poly(n)$ $O(nW)$ is $poly(n)$

pseudo-poly $\rightarrow$

Input size $N = \overset{\text{registers}}{n} + \log W / \log n$ bits to represent W

e.g if $W = 2^n \Rightarrow N = \Theta(n)$ but

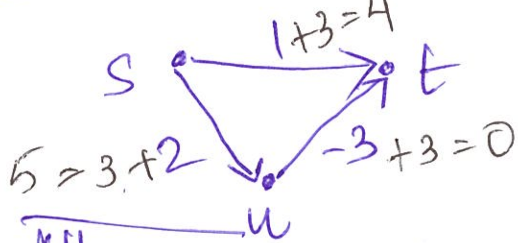
runtime $= n \cdot 2^n = O(N \cdot 2^N)$.

Shortest path problem

Input: (•) Directed graph $G = (V, E)$
$\forall e \in E$, cost c_e ($c_e < 0$ is allowed)
BUT no negative cycle
(•) $t \in V$

Output: $\forall s \in V$, output a shortest s-t path

Attempt 1: Run Dijkstra



Shortest s-t path: s, u, t
Dijksta path: s-t

Attempt 2: Add some $\Delta > 0$ to all edges s.t. the new edge cost $c_e' = c_e + \Delta \geq 0$

Doesn't work: ~~See above example w/ $\Delta = 3$~~ $\Delta = 3$ above
(See T/F #7 on piazza)

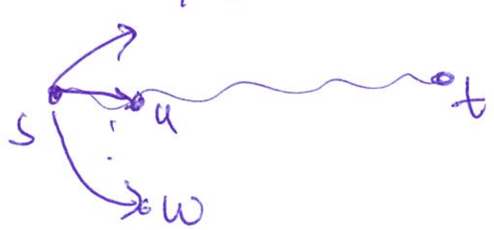
NOTE: No known greedy or divide + conquer algo.

Assume: only are interest in cost of shortest s-t path

Attempt 3: $OPT(s)$ = cost of shortest s-t path. $\forall s \in V$

(Q1) How many sub-problems? n sub-problems

(Q2) Recurrence relation: If a shortest s-t path has (s,u) as its 1st edge
$s \neq t$

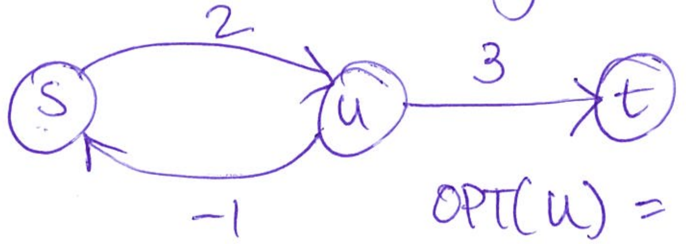


$OPT(s) = c_{(s,u)} + OPT(u)$

More generally

$$OPT(s) = \min_{\substack{w: \\ (s,w) \in E}} \{ c_{(s,w)} + OPT(w) \}$$

(Q3) Total ~~ord~~ ordering among the sub-problem.



$$OPT(s) = \cancel{c_{(s,u)}}^{2} + OPT(u)$$

$$OPT(u) = \min \{ \cancel{c_{(u,s)}}^{-1} + OPT(s), \; \cancel{c_{(u,t)}}_{3} + OPT(t) \}$$

Cyclic dependence among u ~~&~~ & s $\Rightarrow$ X total ordering

<u>So far</u>: In the 2 problems, the sub-problems only used parameter given in the problem definition.

<u>Idea:</u> Introduce an (implicit) parameter in your sub-problems

no present in problem defn

<u>Attempt 4:</u> Lecture from FA22, valid attempt

$\hookrightarrow$ exp. many sub-problem.