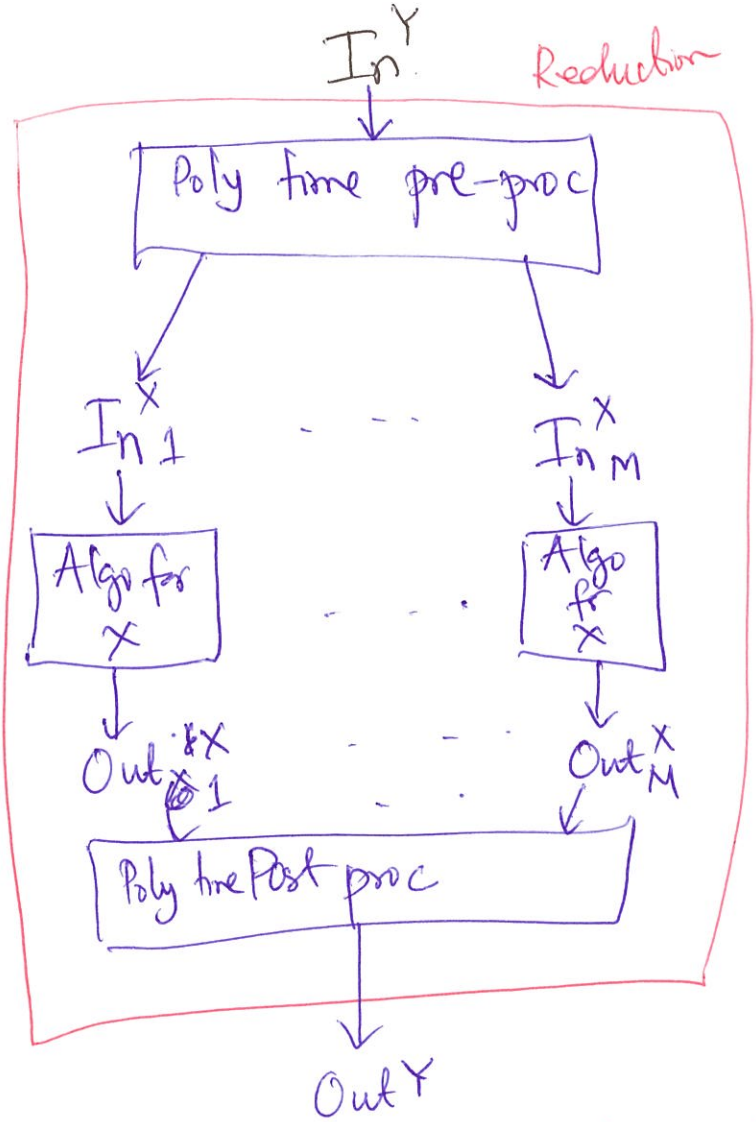
Nov 22

# $Y \leq_p X$

Y is polytime reducible to X

$\Leftrightarrow \exists$ a poly time reduction from Y to X

Solve: $In^Y$ - - - - → $Out^Y$




$M \geq 1$

$M = 1$

Total runtime

$= poly(N)$

$+ M \cdot$ (Runtime for Algo for X)

$+ poly(N)$

Assume: $M \leq poly(N)$

If Algo for X runs in poly(N) time

$\Rightarrow$ Overall: ($\leq poly(N)$)

$= poly(N) + M \cdot poly(N) + poly(N)$

$\leq poly(N)$

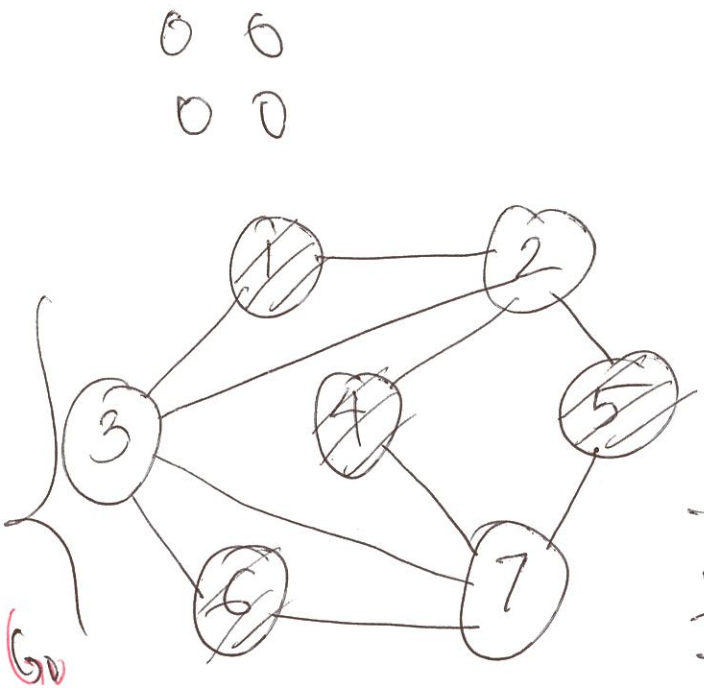
Example: HW2 Q2 $\leq_p$ Stable matching ($M=1$)

Going forward: ONLY consider problems w/ Binary output.

**Problem:** Independant Set (IS) problem

$G = (V, E)$

**Def:** An IS of $G$ is a subset $S \subseteq V$ s.t there is NO edge between ANY distinct pairs of vertices in $S$




$\{1,4\}$ ✓

$\{3,7\}$ ✗ $\{1,4,7\}$ ✗

$\{3,4,5\}$ ✓

$\{1,4,5,6\}$ ✓

**Formal Problem** **decision problm**

**Input:** $G = (V, E)$ $|V| = n$ + $0 \leq k \leq n$

**Output:** TRUE / 1 $\iff$ $G$ has an IS of size $\geq k$

Ex: $G_0, 2$ ✓ $G_0, 3$ ✓ $G_0, 4$ ✓ $G_0, 5$ ✗

**Note:** Subset of an IS is also an IS