Dec 2

RECAP

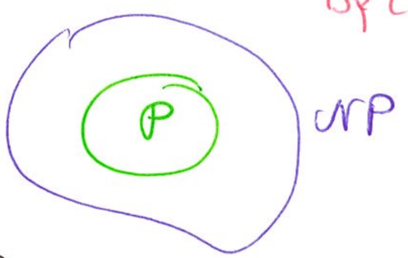
→ Def: $\mathcal{P}$: set of all problems that can be solved by a poly time algo

→ Def: $\mathcal{NP}$: $Y \in \mathcal{NP}$ iff $\exists$ an efficient verifier B_Y s.t $\forall w$:

(i) $w \in Y \Rightarrow \exists$ a witness t with $|t| \le poly(|w|)$ s.t. $B_Y(w,t)=1$

(ii) $w \notin Y \Rightarrow \forall$ witnesses t with $|t| \le poly(|w|)$, $B_Y(w,t)=0$

→ $\mathcal{P} \subseteq \mathcal{NP}$ $(*)$



Pf: $Y \in \mathcal{P}$ - - - - - $Y \in \mathcal{NP}$

$\hookrightarrow \exists$ a poly time algo A s.t $A(w)=1 \Leftrightarrow w \in Y$

Show: $Y \in \mathcal{NP}$ by showing $\exists$ an efficient verifier B_Y

$B_Y(w,t)$:
return $A(w)$

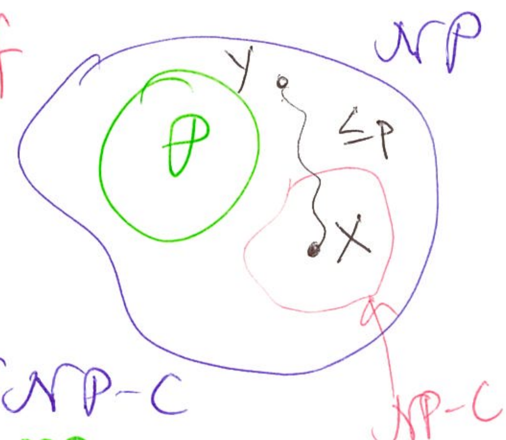
$\Rightarrow \forall$ witness t $B(w,t) = A(w)$

$\Rightarrow Y \in \mathcal{NP}$ ∎

Def: X is $\mathcal{NP}$-complete ($\mathcal{NP}$-C) if

(i) $X \in \mathcal{NP}$

(ii) $\forall Y \in \mathcal{NP}$, $Y \le_p X$



Lem: Let X is $\mathcal{NP}$-C. ~~If~~ $X \in \mathcal{P}$ $\Rightarrow \mathcal{P} = \mathcal{NP}$

Pf: By ~~contradiction~~. Let X is $\mathcal{NP}$-C

Argue: $\underline{\mathcal{NP} \subseteq \mathcal{P}}$ + $(*)$ = $\mathcal{P} = \mathcal{NP}$

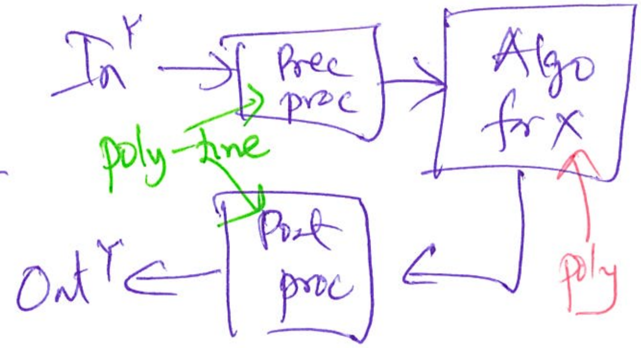
Let $Y \in NP$ ~~If~~ Assume $X \in P$

We know $Y \le_p X$

$\Rightarrow \exists$ a poly time algo for Y (as $X \in P$)

$\Rightarrow Y \in P$

$\Rightarrow NP \subseteq P$ ∎



THM: IS is NP-C.

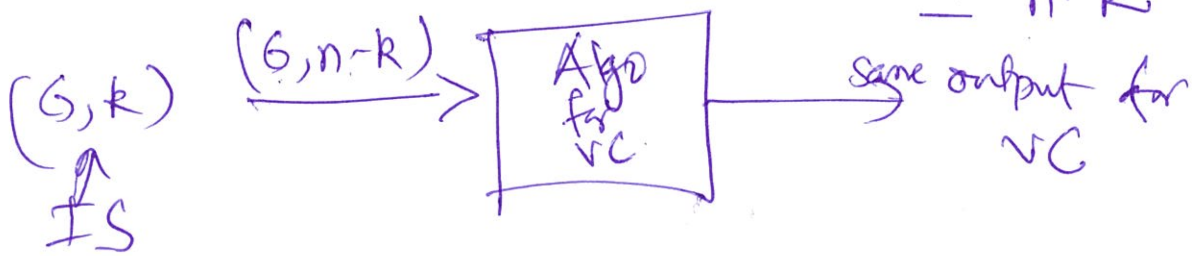
THM: (1) IS $\le_p$ VC (2) VC $\le_p$ IS

Lemma 1: $G = (V, E)$

S is an IS of $G \iff V \setminus S$ is a VC of G

$\iff G$ has an IS of size $\geq k \iff G$ has a VC $\leq n-k$

(1)



(2) Same