Dec 9!

THM 3! 3-coloring is NP-C

THM 4! $\boxed{3\text{-SAT} \le_p 3\text{-coloring}} \le_p k\text{-coloring}$ — see the book; $k \geq 3$

$\Rightarrow$ + 3-SAT is NP-C + 3-colorg $\in$ NP

Goal: $3\text{-SAT} \le_p 3\text{-coloring}$

Pf (idea) Given a 3-SAT formula

$\Phi = C_1, \ldots, C_m$ on $X_1, \ldots, X_n$ $\qquad C_i = t_1 \vee t_2 \vee t_3$

$|\Phi| = \Theta(m+n)$

in poly $(n+m)$ time generate a graph G_Φ

s.t. Φ is satisfiable $\Longleftrightarrow$ G_Φ is 3-colorable

Redux! Algo Sat (Φ)

1. $\boxed{\text{Construct } G_\Phi \text{ from } \Phi}$
2. $b \leftarrow$ Algo-3-color (G_Φ)
3. Return b

Idea! Use gadgets again ("Δ" will be more complicated)

High level idea:

Step 1: Create graph G_0 s.t. any valid 3-coloring of G_0 $\equiv$ an assignment to $X_1, \ldots, X_n$

Step 2: For each clause C_i, create a graph G_i $\rightarrow$ ~~G_0~~ $G_\Phi = G_0 \cup G_1 \cup \cdots \cup G_m$

G_Φ

s.t. a 3 colorg of G_Φ $\equiv$ a satisfying assignment to Φ.

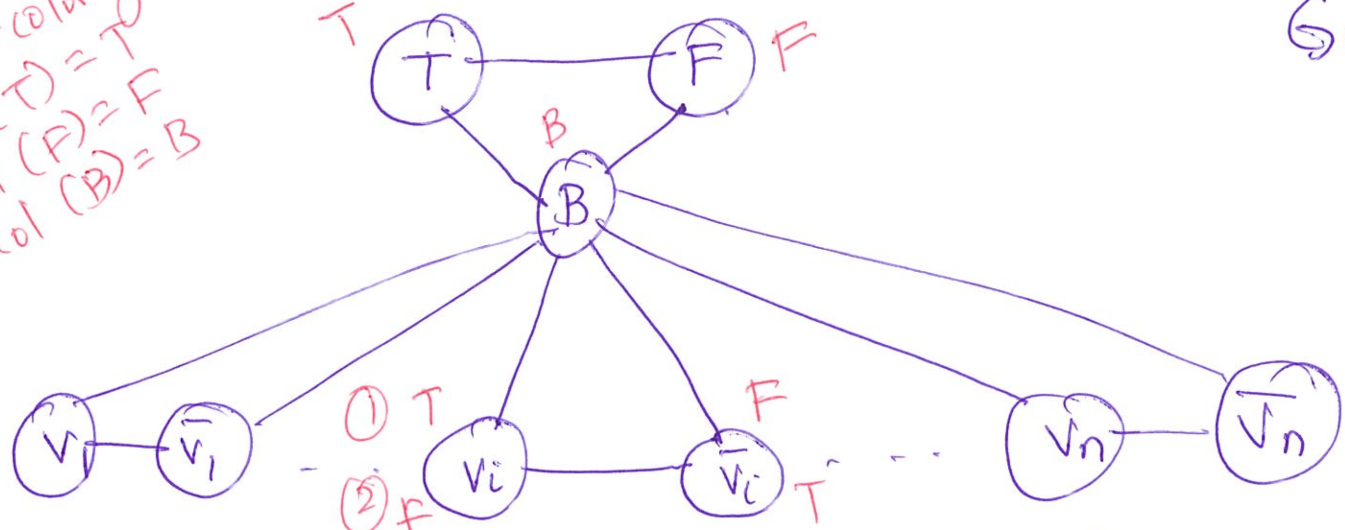
Step 1: G_0 $2n+3$ vertices $v_1, \dots, v_n$ $v_i \equiv x_i$

$\bar{v}_1, \dots, \bar{v}_n$ $\bar{v}_i \equiv \bar{x}_i$

3 special nodes T, F, B

∃ a valid 3-coloring
col(T) = T
col(F) = F
col(B) = B

G_0



Claim: ∃ a valid 3-coloring of G_0 ≡ an assignment to $x_1, \dots, x_n$

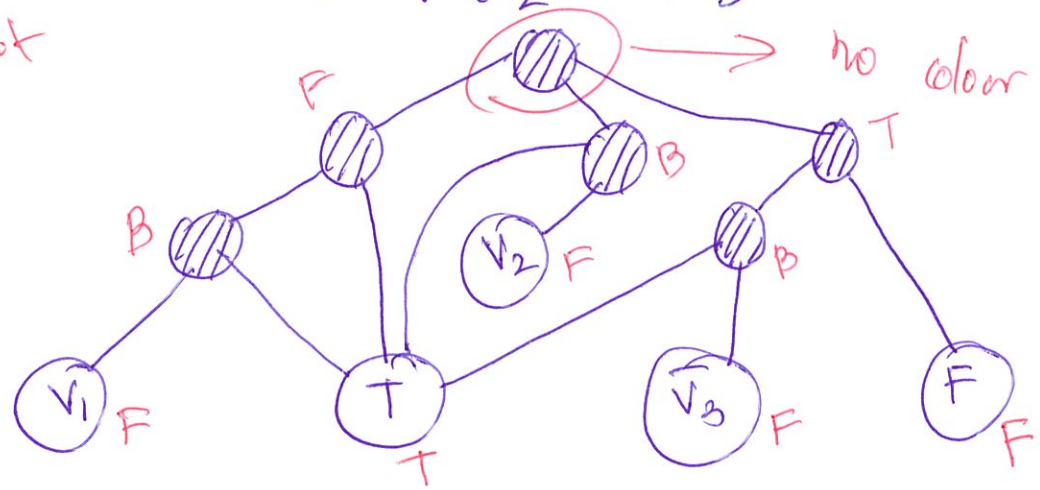
$\forall i$ either (i) $c(v_i) = T$ and $c(\bar{v}_i) = F$

OR (ii) $c(v_i) = F$ and $c(\bar{v}_i) = T$

Step 2: Encode each clause C_i with a graph/gadget G_i

$C_i = x_1 \vee x_2 \vee \bar{x}_3$

→ unique to G_i



no colour remains for this

If not

Claim: In a valid 3-coloring of $G \equiv \{G_i + G_0\}$ at least one of v_1 or v_2 or $\bar{v}_3$ is colored T

t_1 t_2 t_3

In general $C_i = t_1 \vee t_2 \vee t_3$

Final exam material.