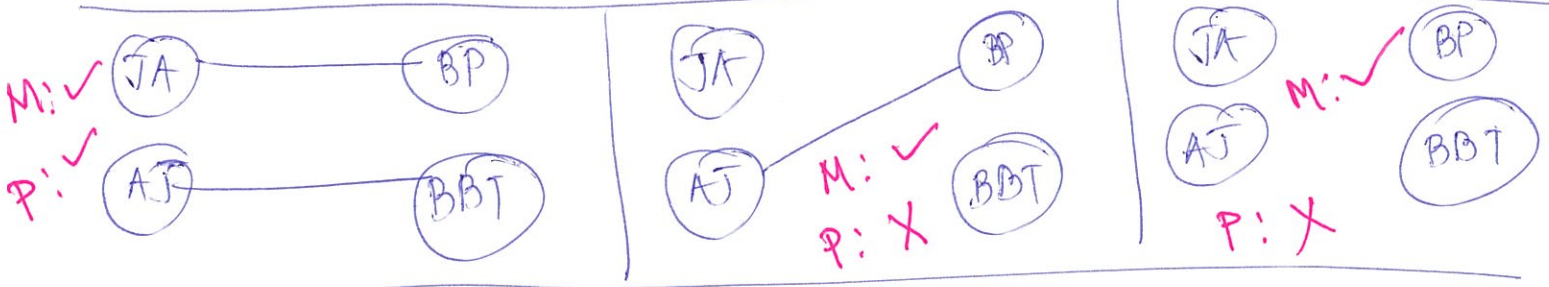
Sep 6

Stable matching / marriage problem | n = 2

n men $M = \{m_1, \dots, m_n\}$ | $M = \{BP, BBT\}$

n women $W = \{w_1, \dots, w_n\}$ | $W = \{JA, AJ\}$



Def (Matching) Matching S is a subset of pairs of women and men

such that (s.t.)

$$S \subseteq W \times M \overset{\text{def}}{=} \{(w,m) \mid w \in W,\ m \in M\}$$

(i) $\forall\ w \in W$ (for all, in), $\exists$ (there exists) at most one $m \in M$ s.t. $(w,m) \in S$

AND EXACTLY

(ii) $\forall\ m \in M$, $\exists$ at most one EXACTLY $w \in W$ s.t. $(w,m) \in S$

Def: Perfect matching ↓

(JA, BP) ← pair/tuple

pair ← a set of pairs

{(JA, BP)} ← Set with a pair ← matching

Matching is a set of pairs.

Preference lists n = 2

L_{JA} : BP > BBT

L_{AJ} : BP > BBT

L_{BP} : AJ > JA

L_{BBT} : AJ > JA

Def Preference list:

$\forall w \in W$, L_w: Total ranking/ordering of all men

$\forall m \in M$, L_m: ~~Total~~ Total ranking/ordering of all women

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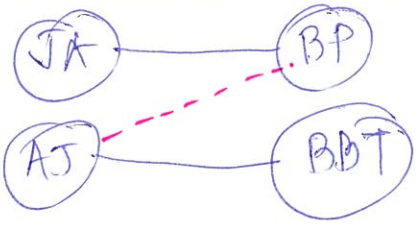

Def: A stable matching S is

(1) S is a perfect matching AND

(2) It has no instability

Def: (Instability) Given 2n pref lists AND a perfect matching & $S \subseteq W \times M$,

a pair $(w', m') \notin S$ is an instability

(not in)

IF

(i) $m' > m$ in $L_{w'}$

AND

(ii) $w' > w$ in $L_{m'}$