

Step 9

Gale-Shapley algo

Running Example

$n = 2, W = \{JA, AJ\}, M = \{BP, BBT\}$

$L_{JA}: BP > BBT$ $L_{BP}: JA > AJ$
 $L_{AJ}: BBT > BP$ $L_{BBT}: AJ > JA$

JA	AJ	BP	BBT
(F)	F	F	F

- ① Initialize all n men and n women as free in book men propose
- ① In a loop: A free woman proposes to a man
- ... **stuff happens** ...
- ② We have n matched pairs

Q1 Who should JA propose to?

A1 BP
 JA $\xrightarrow{\text{proposes to}}$ BP

Q2 What should BP do?

Accept?
 Reject? **(JA, BP) get engaged**

JA	AJ	BP	BBT
E	F	E	F

AJ is free

Q3 Who should AJ propose to?

A3 BBT
 AJ $\xrightarrow{\text{proposes to}}$ BBT

Q4 What should BBT do?

(AJ, BBT) get engaged

Q4 What should m do?

Case 2.1 m is free
 (w, m) get engaged

Case 2.2 (w', m) are engaged

Case 2.2.1 $w' > w$ in L_m . nothing

Case 2.2.2 $w > w'$ in L_m . w is free

Initial state: All n men + women are free

- ① Let w be a free woman
- Q1 Who should w propose to?
- A1 w proposes to man m on top of her pref list (L_w)
- $\hookrightarrow w$ proposes to m
- Q2 What should m do?
- Accept \rightarrow might get a better proposal later
- Rejects \rightarrow might not get a better proposal later
- $\hookrightarrow (w, m)$ get engaged

General state: each man/woman is F or E

Case 1: All n men & n women are engaged \rightarrow Algo terminates & outputs the n engaged pairs

Case 2: If a free woman w

- Q3 Who should w propose to?
- A3 We will pick the best man m that w has NOT proposed to yet.
- $\hookrightarrow w$ proposes to m