Sep 13 | Obs 0 S is a matching.

Obs 1 Once m gets engaged, he keeps getting to better women

Obs 2 If w proposes to m' after m, $m > m'$ in L_w

Lemma 0 GS terminates in $\leq n^2$ iterations ✓

Lemma 1 S is a perfect matching

Lemma 2 S has no instability

Lemmas 0+1+2 ⇒ GS always outputs a stable matching

Lemma 3 If at the end of any iteration, w is free
⇒ w has NOT proposed to ALL men
↑ This was missing in notes from Wed

Assuming Lemma 3 is true, we'll prove Lemma 1.

Pf (idea) of Lem 1: S is a perfect matching

Pf by contradiction (Use Obs 0, Lem 0+3, Algo definition)

Pf details: For the sake of contradiction, assume S is not a perfect matching.

⇒ ∃ a woman w who is not engaged

by Obs 0, algo def. ⇒ ∃ a free woman w

⇒ w has not proposed to ALL men

Lem 3 ⇒ ∃ a free woman w who has not proposed to all men

⇒ GS Algo has not terminated.

Algo def → contradicts Lem 0 ∎

Pigeon hole principle (PHP) If $\leq n-1$ pigeons are placed in n holes, $\Rightarrow \exists$ an empty hole.

Pf (idea) of Lem 3: Pf by contradiction (Use Obs 1, PHP, Algo Def)

Pf (details) $\neg(P \Rightarrow Q) = P \wedge \neg Q$ ↖

$\equiv \neg P \vee Q$ $\neg(\neg P \vee Q)$

Assume w is free AND w has proposed to all men

$\Rightarrow$ all men have received atleast 1 proposal

$\Rightarrow$ all n men are engaged ——— (*)

Algo def + Obs 1

Since w is free $\Rightarrow \leq n-1$ women are engaged

$\Rightarrow \geq 1$ man is free

PHP

hole :: man

pigeon :: woman

$\Rightarrow \leq n-1$ men are engaged

Algo def. ↪ contradicts (*) ∎

Case 2.2.1 when m rejects w' proposal. ((w'', m) engaged)

$w'' > w'$ in Lm

$w > w''$ in Lm (Obs 1)

transitivity $\Rightarrow$ $w > w'$ in Lm

Q: What is the total size / space needed to represent all prefs