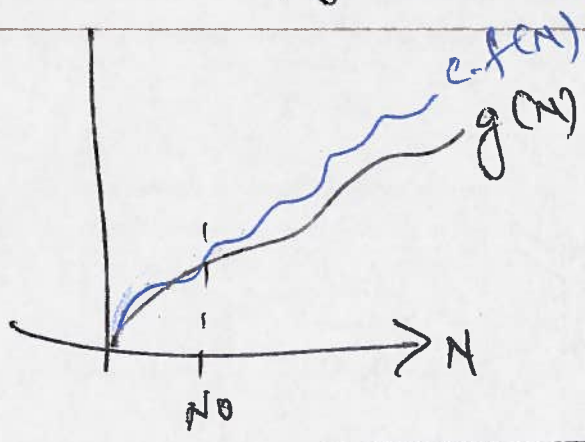


Sep 13

### Big-O notation:

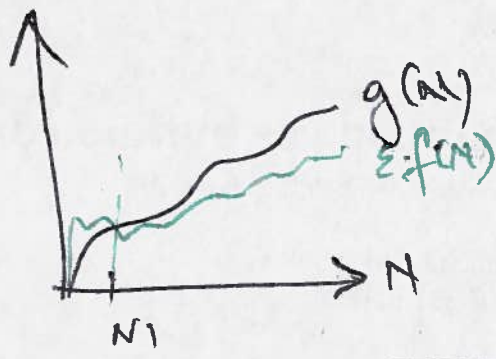
$g(N)$  is  $O(f(N))$  if



$\exists c > 0, N_0 > 0$

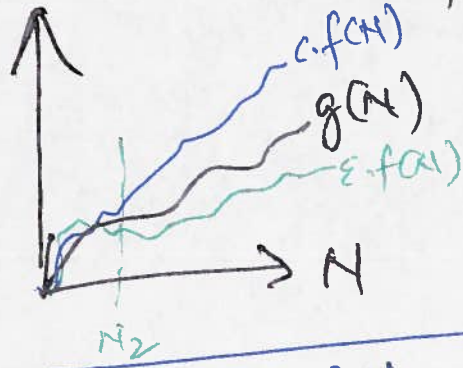
$|0N+1| \stackrel{?}{=} O(\sqrt{N}) \times$   
 $= O(N) \checkmark$   
 $= O(N^2) \checkmark$   
 $= O(N!) \checkmark$

### Big-Omega notation: $g(N)$ if $\Omega(f(N))$ if $\exists \epsilon > 0, N_1 > 0$



$|0N+1| \stackrel{?}{=} \Omega(\sqrt{N}) \checkmark$   
 $= \Omega(N) \checkmark$   
 $= \Omega(N^2) \times$   
 $= \Omega(N!) \times$

### Big-Theta notation: $g(N)$ is $\Theta(f(N))$ if (1) $g(N)$ is $O(f(N))$ AND (2) $g(N)$ is $\Omega(f(N))$



$|0N+1| \stackrel{?}{=} \Theta(\sqrt{N}) \times$   
 $= \Theta(N) \checkmark$   
 $= \Theta(N^2) \times = \Theta(N!) \times$

### Properties of O: (Also valid for $\Omega$ and $\Theta$ )

- ①  $g(N)$  if  $O(f(N))$  and  $f(N)$  is  $O(h(N)) \Rightarrow g(N)$  is  $O(h(N))$
- Ex:  $|0N+1|$  is  $O(N)$  &  $N$  is  $O(N^2)$   
 $\Rightarrow |0N+1|$  is  $O(N^2)$

② Additive:  $f(N)$  is  $O(h(N))$  &  $g(N)$  is  $O(h(N))$   
 $\Rightarrow f(N) + g(N)$  is  $O(h(N))$

Ex.  $10N$  is  $O(N)$  &  $1$  is  $O(N)$   $\Rightarrow 10N + 1$  is  $O(N)$

③ Multiplicative  $f(N)$  is  $O(h_1(N))$  &  $g(N)$  is  
 $O(h_2(N)) \Rightarrow f(N) \cdot g(N)$  is  $O(h_1(N) \cdot h_2(N))$

Ex.  $N$  is  $O(N)$  &  $10$  is  $O(1)$   $\Rightarrow 10N$  is  $O(N)$