Markov Decision Process

Lecture 2.1

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CSE4/510 Reinforcement Learning
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*Slides has been modified from David Silver’s RL course
Overview

1 Learning

2 Definition

3 Markov Decision Processes (MDP)
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3 Markov Decision Processes (MDP)
Why do we need to learn?
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There are (at least) two distinct reasons to learn:

1. Find previously unknown solutions. E.g., a program that can play Go better than any human, ever

Reinforcement learning seeks to provide algorithms for both cases.

Note that the second point is not (just) about generalization — it is about learning efficiently online, during operation.
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2. Find solutions online, for unforeseen circumstances. E.g., a robot that can navigate terrains that differ greatly from any expected terrain
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Why do we need to learn?

Science of learning to make decisions from interaction. This requires us to think about:

- ...time
- ...(long-term) consequences of actions
- ...actively gathering experience
- ...predicting the future
- ...dealing with uncertainty
Examples of decision problems

Examples:

- Fly a helicopter
- Manage an investment portfolio
- Control a power station
- Make a robot walk
- Play video or board games

These are all reinforcement learning problems (no matter which solution method you use)
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Core concepts

Core concepts of a reinforcement learning system are:

- Environment
Core concepts

Core concepts of a reinforcement learning system are:

- Environment
- Reward signal
Core concepts of a reinforcement learning system are:

- Environment
- Reward signal
- Agent, containing:
  - Agent state
  - Policy
  - Value function (probably)
  - Model (optionally)
The **agent** is acting in an **environment**. How the environment reacts to certain actions is defined by a **model** which we may or may not know. The agent can stay in one of many **states** \((s \in S)\) of the environment, and choose to take one of many **actions** \((a \in A)\) to switch from one state to another. Which state the agent will arrive in is decided by the **transition probabilities** between states \(P(s'|s, a)\). Once an action is taken, the environment delivers a **reward** \((r \in R)\) as a feedback.
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At each step $t$ the agent:

- Receives state $S_t$ / observation $O_t$ and reward $R_t$
- Executes action $A_t$

The environment:

- Receives action $A_t$
- Emits state $S_{t+1}$ / observation $O_{t+1}$ and reward $R_{t+1}$
Finite Markov Decision Processes (MDP)

Markov property:

$$P[S_{t+1}|S_t] = P[S_{t+1}|S_1, S_2, \ldots, S_t]$$
Finite Markov Decision Processes (MDP)

Markov property:

\[ P[S_{t+1}|S_t] = P[S_{t+1}|S_1, S_2, \ldots, S_t] \]

“The future is independent of the past given the present”

Daily life trajectory:

\[ S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots, S_T \]
A Markov Chain is a tuple $\langle S, P \rangle$
- $S$ is a set of states
- $P$ is a state transition probability matrix

$$P_{ss'} = P[S_{t+1} = s'|S_t = s] \quad (1)$$
Sample episodes for Student Markov Chain starting from $S_1 = C1$.

Episodes:

- C1 C2 C3 Pass Sleep
- C1 FB FB C1 C2 Sleep
- C1 C2 C3 Pub C2 C3 Pass Sleep
- C1 FB FB C1 C2 C3 Pub C1 FB FB FB C1 C2 C3 Pub C2 Sleep
Markov Chain - Student Example

\[ P = \begin{bmatrix}
    0.5 & 0.8 & 0.6 & 0.4 \\
    0.2 & 0.4 & 0.4 & 0.1 \\
    0.4 & 0.4 & 0.4 & 0.9 \\
\end{bmatrix} \]
Markov reward process is a Markov chain with values.

**Definition**

A Markov Reward Process is a tuple \( \langle S, P, R, \gamma \rangle \)

- \( S \) is a set of states
- \( P \) is a state transition probability matrix
  
  \[
  P_{ss'} = \mathbb{P}[S_{t+1} = s' \mid S_t = s]
  \]  
  (2)

- \( R \) is a reward function, \( R_s = \mathbb{E}[R_{t+1} \mid S_t = s] \)
- \( \gamma \) is a discount factor, \( \gamma \in [0, 1) \)
Discount Factor $\gamma$

The discounting factor $\gamma \in [0, 1)$ penalize the rewards in the future. Reward at time $k$ worth only $\gamma^{k-1}$

Motivation:

- The future rewards may have higher uncertainty (stock market)
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- The future rewards do not provide immediate benefits (As human beings, we might prefer to have fun today rather than 5 years later ;)
- Discounting provides mathematical convenience (we don’t need to track future steps infinitely to compute return)
- It is sometimes possible to use undiscounted Markov reward processes (i.e. $\gamma = 1$) e.g. if all sequences terminate.
Return $G_t$

**Definition**

The *return* $G_t$ is the total discounted reward from time-step $t$.

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- $\gamma$ is a discount factor ($\gamma \in [0, 1]$)
- $R$ is the reward
- The value of receiving reward $R$ after $k + 1$ time-steps is $\gamma^k R$
Sample returns for Student MRP:
Starting from $S_1 = C1$ with $\gamma = 0.5$

$$G_1 = R_2 + \gamma R_3 + \cdots + \gamma^{T-2} R_T$$
Sample **returns** for Student MRP:
Starting from $S_1 = C1$ with $\gamma = 0.5$

$$G_1 = R_2 + \gamma R_3 + \cdots + \gamma^{T-2} R_T$$

<table>
<thead>
<tr>
<th>State Sequence</th>
<th>$v_1$ Calculation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 C2 C3 Pass Sleep</td>
<td>$v_1 = -2 - 2 \cdot \frac{1}{2} - 2 \cdot \frac{1}{4} + 10 \cdot \frac{1}{8}$</td>
<td>$-2.25$</td>
</tr>
<tr>
<td>C1 FB FB C1 C2 Sleep</td>
<td>$v_1 = -2 - 1 \cdot \frac{1}{2} - 1 \cdot \frac{1}{4} - 2 \cdot \frac{1}{8} - 2 \cdot \frac{1}{16}$</td>
<td>$-3.125$</td>
</tr>
<tr>
<td>C1 C2 C3 Pub C2 C3 Pass Sleep</td>
<td>$v_1 = -2 - 2 \cdot \frac{1}{2} - 2 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} - 2 \cdot \frac{1}{16} \ldots$</td>
<td>$-3.41$</td>
</tr>
<tr>
<td>C1 FB FB C1 C2 C3 Pub C1 …</td>
<td>$v_1 = -2 - 1 \cdot \frac{1}{2} - 1 \cdot \frac{1}{4} - 2 \cdot \frac{1}{8} - 2 \cdot \frac{1}{16} \ldots$</td>
<td>$-3.20$</td>
</tr>
<tr>
<td>FB FB FB C1 C2 C3 Pub C2 Sleep</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Markov Decision Process (MDP)

Definition

A Markov Decision Process (MDP) is a tuple \( \langle S, A, P, R, \gamma \rangle \)

- S is a set of states
- A is a set of actions
- P is a state transition probability matrix

\[
P_{ss'} = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]
\]  

- R is a reward function, \( R_s = \mathbb{E}[R_{t+1} | S_t = s, A_t = a] \)
- \( \gamma \) is a discount factor, \( \gamma \in [0, 1) \)
Mains reasons to learn (not just for agents) are to find previously unknown solutions and to the find solutions in unforeseen circumstances.

Core parts of a reinforcement learning are: Environment, Reward, Agent.

Markov property: The future is independent of the past given the present.

The discounting factor $\gamma \in [0, 1)$ penalize the rewards in the future.

Markov Decision Process (MDP) defined as a tuple $\langle S, A, P, R, \gamma \rangle$. 