

# Group Equivariant Q- networks

Kiran Vaddi

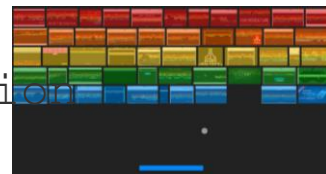
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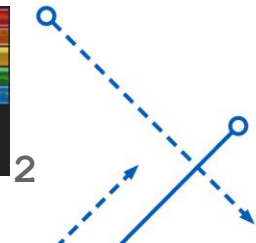
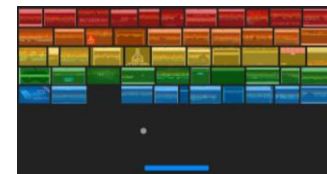
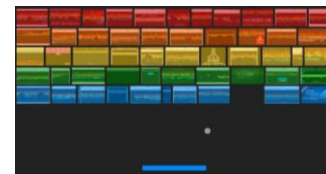
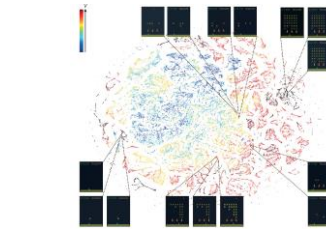
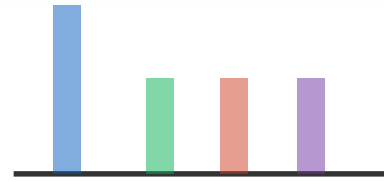
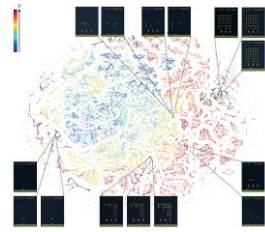
Value function  
Approximators



What the emulator sees

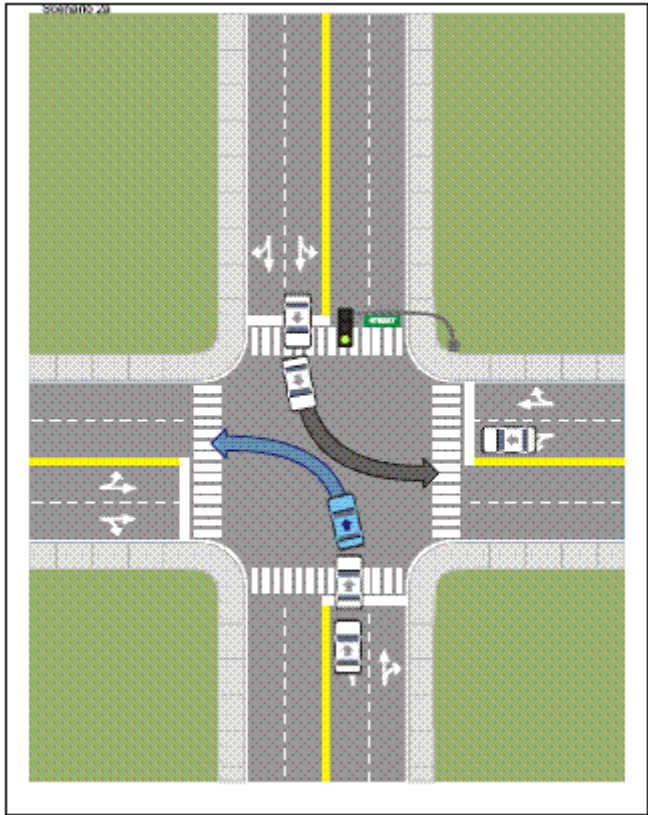


State Representation



# Why do we need equivariance?

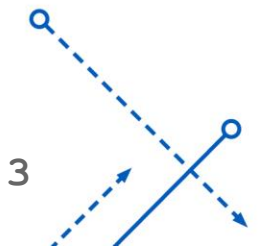
Blue and Black lines are symmetric



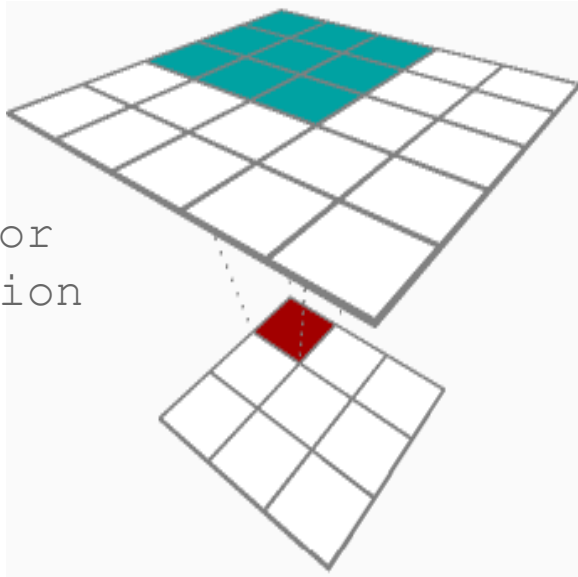
Agent can better estimate consequences:

- considers what it would if its on the symmetrically opposite path
- Doesn't have to re-learn

- Nature by default has some symmetry encoded into it
- Learning a symmetry valued representation is a better model of the environment
- Value function approximation learns a simple representation of model to make decision (refer t-sne plot in original DQN paper)



## CNN

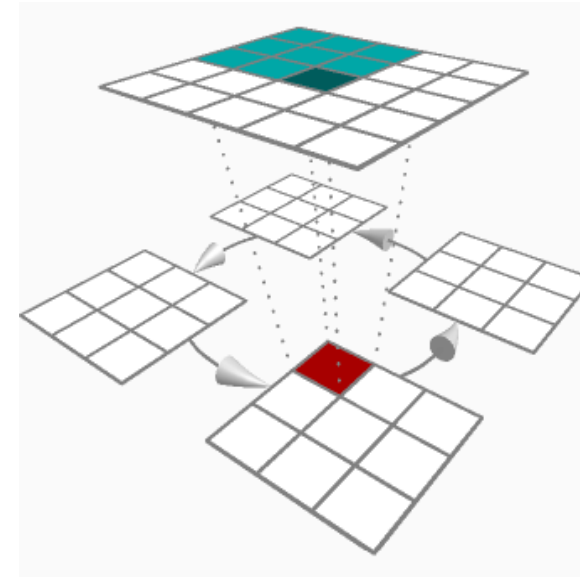


One channel for each translation of filter

$$\text{conv2d}\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

Features are different for bar when rotated

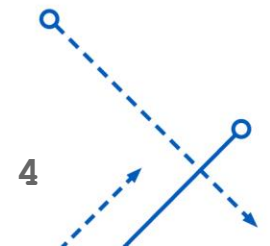
## P4-CNN



One channel for each rotation + translation of filter

$$\text{p4-conv2d}\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Features are same when rotated



For any  $(u, v) \in \mathbb{Z}/2\mathbb{Z}$

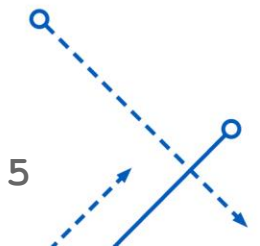
Compute augmented filter bank:

$$g(r, u, v) = \begin{bmatrix} \cos(r\pi/2) & -\sin(r\pi/2) & u \\ \sin(r\pi/2) & \cos(r\pi/2) & v \\ 0 & 0 & 1 \end{bmatrix}$$

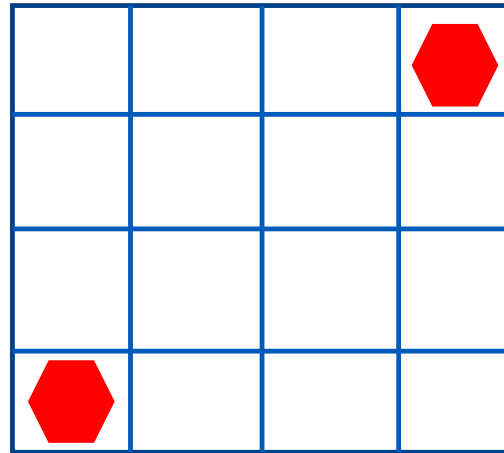
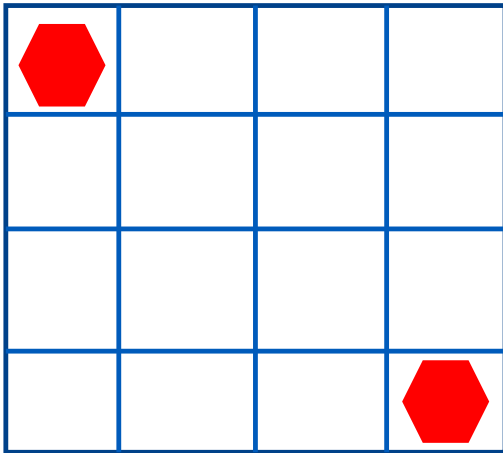
for  $r \in \{0, 1, 2, 3\}$

Now compute convolution for augmented filter bank and store it in corresponding dimension

For 4x84x84 channel image input, you get 4xKxNxN kernel as output

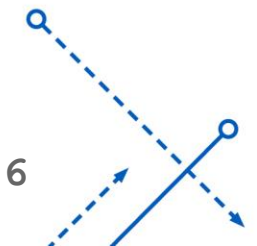


- State representations are important for MDP formulation
- Equivariant representations are important for Model free RL

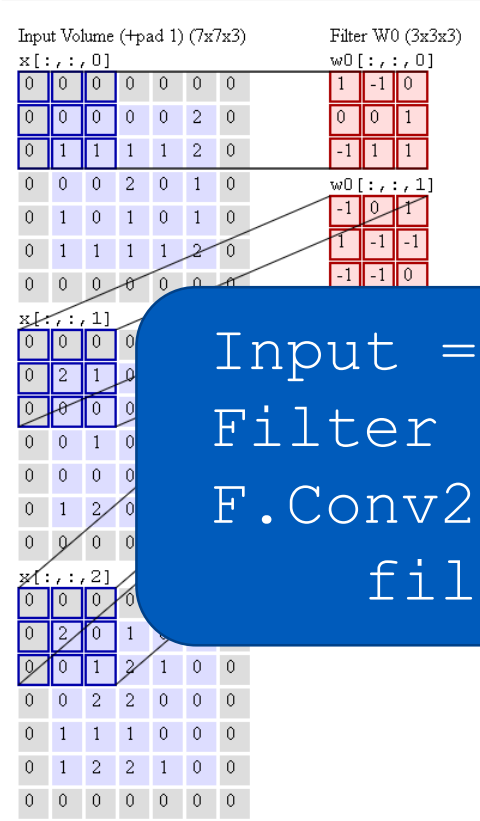


- CNN : Get to the goal states in **left and right** corners
- G-CNN : get to goal states in any of the **opposite** corner

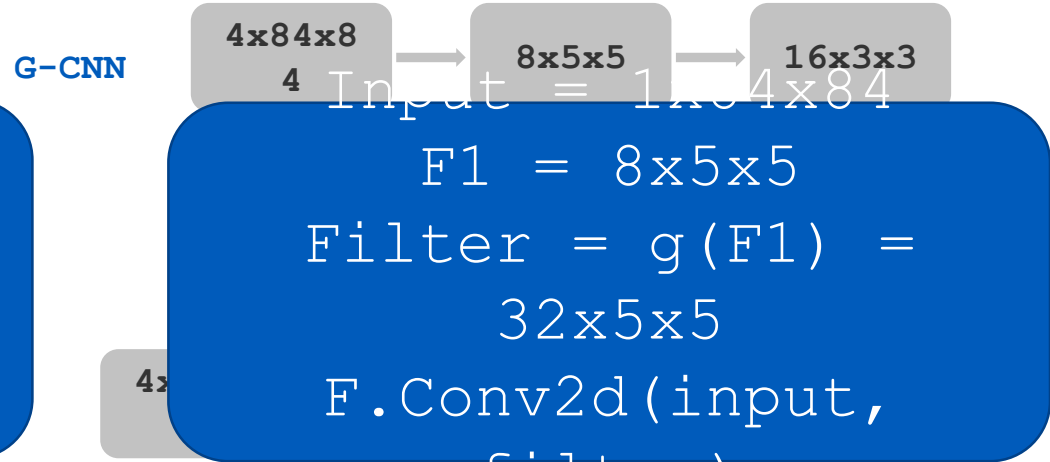
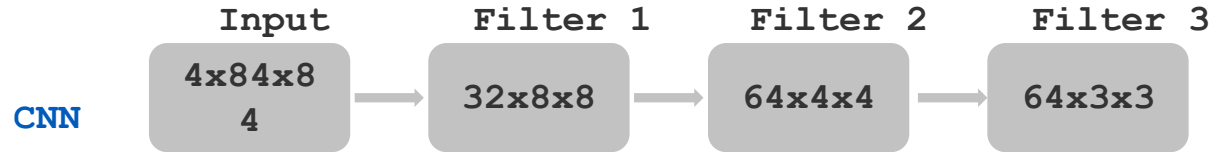
- G-CNN has same representation for both grid worlds



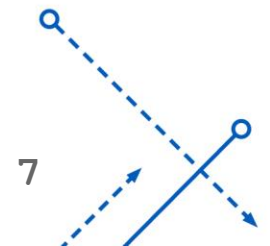
# CNN vs G-CNN structure



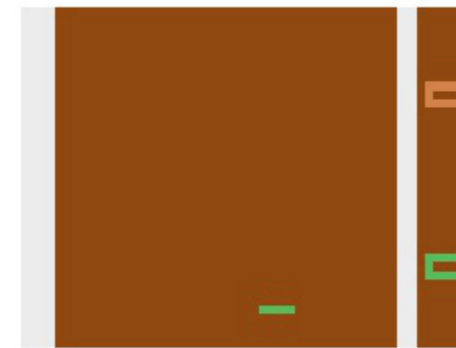
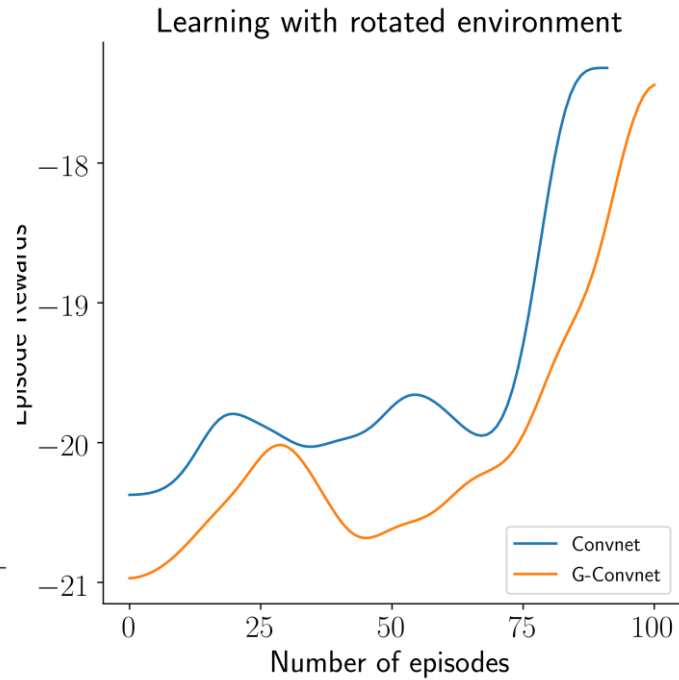
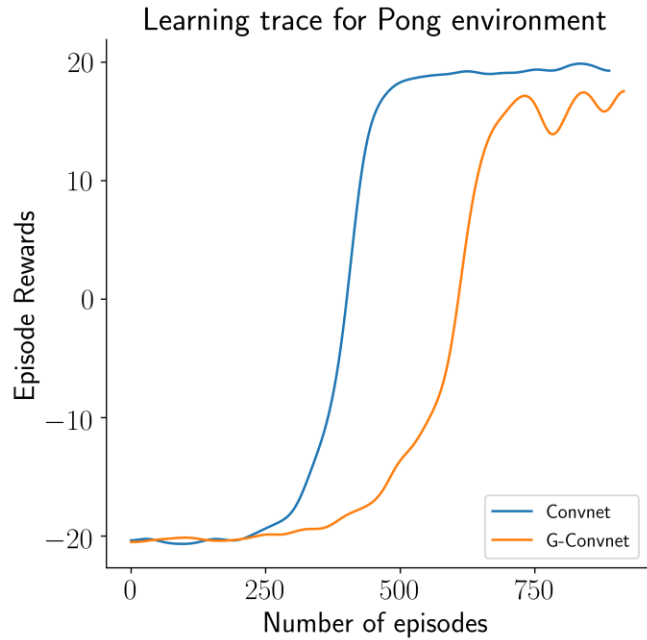
Input = 1x84x84  
 Filter = 32x8x8  
 F.Conv2d(input, filter)



A convolution is computed by convolving filter channel by channel



# Results on Atari Pong

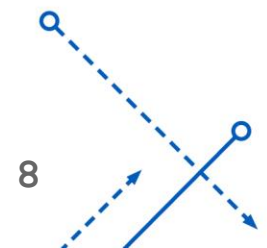


CNN



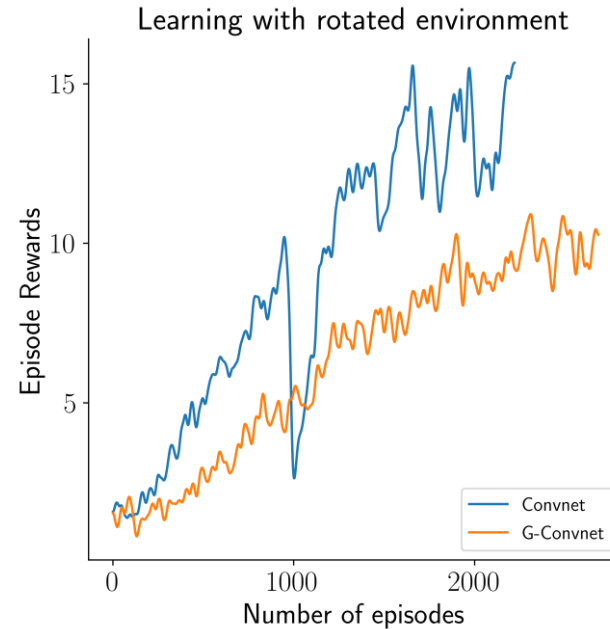
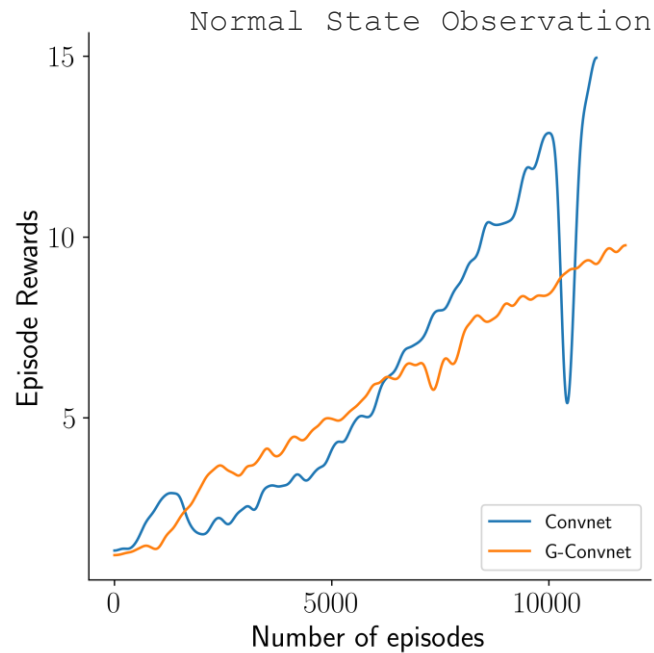
G-CNN

G-convnet can learn using a much shallower network



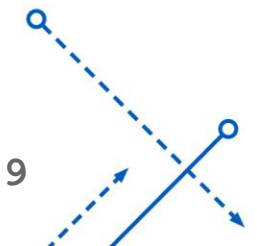


# Results on Atari Breakout



- From DQN paper : “The average total reward metric tends to be very noisy because the distribution of states changes with the policy visits”
- Equivariance in learned filters can be useful in obtaining a better state distribution thus less noisy total reward metric

Learning trace on Atari breakout



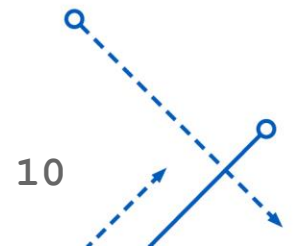
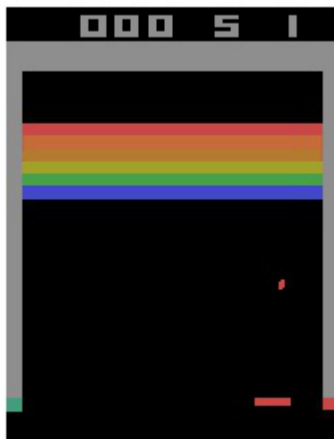
10 M frames

1 M frames

Convnet



G-Convnet



The background of the slide is a complex network of blue lines and arrows. It features several solid blue lines with arrowheads pointing in various directions, some parallel and some intersecting. Interspersed among these are dashed blue lines, some of which form loops or curves. Small open circles are placed at various points along the lines, and some lines terminate in arrowheads. The overall pattern is abstract and technical, suggesting a network or a flow diagram.

# Thank you

Questions?

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