

Topics for mid-term exam: Sample questions: closed book

Topic	Section	material
Number system	1.1-1.6, Hwk1	Radix conversion; 2's complement & signed arithmetic
Boolean algebra	2.4-2.8, Hwk2	Sum of products; Simplify the function to minimum number of literals
Complete problem statement to gate implementation	Hwk3	Implementation of logic expression using NAND gates
Karnaugh maps	3.2-3.6 Up to p.94; 103,104 Hwk3	3 and 4 variable maps, don't cares (x's); XOR
MSI: Decoder and Multiplexers	4.9, 4.11 up to p.162	

1. (4 X 5 = 20 points) Number system and Radix Conversion

For the numbers given below convert the radix as specified.

- 1234 decimal to binary $(1234)_{10} \rightarrow (?)_2$
- 1011.11 to decimal $(1011.11)_2 \rightarrow (?)_{10}$
- 1011.11 to octal $(1011.11)_2 \rightarrow (?)_8$
- 1011.11 to hexadecimal $(1011.11)_2 \rightarrow (?)_{16}$

2. (20 points) Boolean Algebraic Simplification

Simplify using only Boolean algebraic laws and theorems. Clearly show all the intermediate steps. Provide the result in sum of products form.

$$F(A, B, C) = A.B.C + A'.B.C + A'.B.C' + A.B'.C + A.C'$$

3. (4+ 2 + 9 + 5 = 20 points) Word Problem to Gate implementation

Consider a 4-input (W, X, Y, Z) and 1-output function that has logic-1 output whenever the majority of the inputs are logic-0.

- Draw the truth table representing this function.
- Express the function in sum of minterms format.
- Simply the expression from above using algebraic simplification method.
- Draw the combinational circuit for the simplified expression.

4. (10 points) NAND only implementations

Draw the NAND only implementations for the Boolean expression given below:

$$F(A, B, C, D) = A.B + B'.C' + B.(C'+D)$$

5. (20 points) Signed Binary Arithmetic

Consider 8-bit binary containers with 1 bit for sign and 7 bits for magnitude. Consider numbers $A = 65$ and $B = 72$. Assume negative numbers are represented as 2's complement and the operations are in 2's complement. Perform the operations below in binary. Specify if the result is positive, negative or overflow and explain your answer.

- $X = A + B$
- $Y = A - B$
- $Z = -A - B$
- $W = -A + B$

6. (10 points) K-map with don't care conditions; simplify and implement using {and, or, not}

$$F(w, x, y, z) = \sum(0, 2, 4, 6, 10, 14) \quad d(w, x, y, z) = \sum(3, 8, 9, 11, 12, 13, 15)$$

7. a. Implement function in question 6 using 4X1 MUX. B. Implement $F(a, b, c) = \sum(0, 2, 4, 6)$ using 3X8 decoder.

Midterm Review

①
March 8, 2017

① a) $(1234)_{10} \rightarrow (?)_2$ radix 2

$$\begin{array}{r}
 2 \overline{)1234} \\
 \underline{2 \ 617} \quad - 0 \\
 2 \overline{)308} \quad - 1 \\
 \underline{2 \ 154} \quad - 0 \\
 2 \overline{)77} \quad - 0 \\
 \underline{2 \ 38} \quad - 1 \\
 2 \overline{)19} \quad - 0 \\
 \underline{2 \ 9} \quad - 1 \\
 2 \overline{)4} \quad - 1 \\
 \underline{2 \ 2} \quad - 0 \\
 \underline{1} \quad - 0 \\
 =
 \end{array}$$

$$(10011010010)_2$$

b)

$(1011.11)_2 \rightarrow (?)_{10}$ decimal

$$\begin{array}{l}
 \downarrow \\
 1 \times 2^0 \rightarrow 1 \\
 \rightarrow 1 \times 2^1 \rightarrow 2 \\
 \rightarrow 0 \times 2^2 \rightarrow 0 \\
 \rightarrow 1 \times 2^3 \rightarrow 8 \\
 \hline
 11 \\
 \text{decimal}
 \end{array}$$

$$\begin{array}{l}
 \downarrow \quad \downarrow \\
 \frac{1}{2^1} + \frac{1}{2^2} \\
 = 0.5 + 0.25 \\
 = 0.75 \\
 \text{decimal}
 \end{array}$$

$$(11.75)_{10}$$

$$\dots | 2^5 | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 | \cdot | 2^{-1} | 2^{-2} | 2^{-3} | 2^{-4} | \dots$$

positional weights of location

March 2, 2017

$$(1101.11)_3 \text{ to } (\quad)_8 ?$$

$$(\quad)_{10}$$

Convert any radix to decimal
then from decimal to Octal

$$(11.75)_{10}$$

base 10 \rightarrow base 8

$$\begin{array}{r} 8 \overline{) 11} \\ \underline{8} \\ 3 \end{array} \quad (13)_8$$

$$\begin{array}{r} 0.75 \times 8 \\ \hline 6 \\ \underline{} \\ 00 \end{array} \quad (13.6)_8$$

d) Hexadecimal is mostly
~~used~~ for representation.

75FFDE

Red green blue

I binary \leftrightarrow Decimal (fraction: Yes)
II Decimal \leftrightarrow any radix (fraction: No)

Math 8, 2017

(3)

z.

$$\begin{aligned} F(A, B, C) &= ABC + A'BC + A'BC' + AB'C + AC' \\ &= ABC + A'BC + A'BC' + AB'C + AC'(B+B') \\ &= \underline{ABC} + A'BC + A'BC' + AB'C + \underline{ABC'} + AB'C' \\ &= AB(C+C') + A'B(C+C') + AB'(C+C') \end{aligned}$$

$$\begin{aligned} &= AB + A'B + AB' \\ &= (A+A')B + AB' \\ &= B + A \cdot B' \\ &= (B+A) \cdot (B+B') = (B+A) \end{aligned}$$

(4)

March 8,
2017

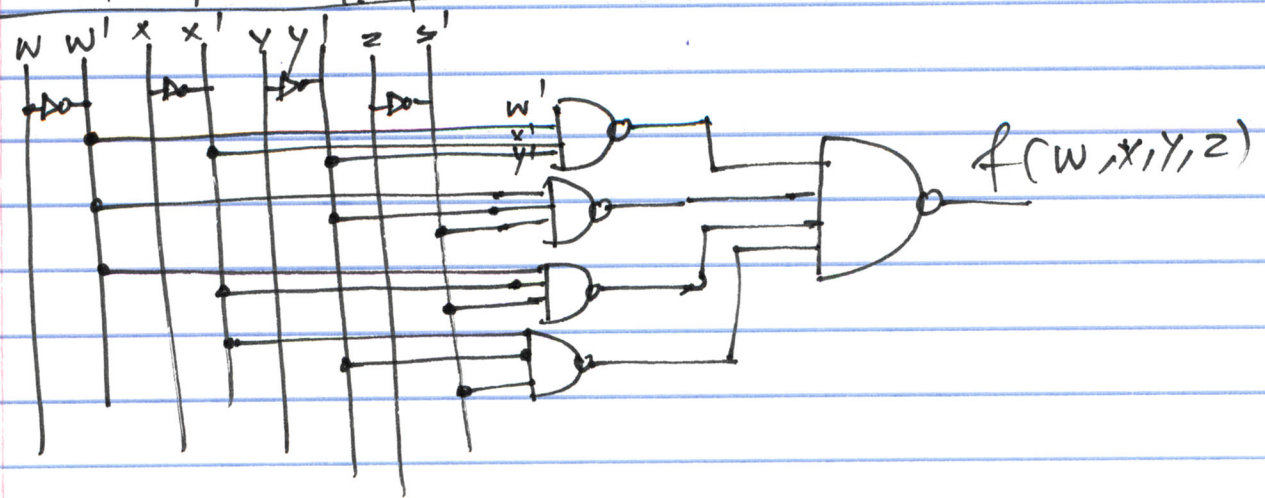
3. Truth Table

W	x	y	z	$f(w, x, y, z)$
0	0	0	0	1 m_0
0	0	0	1	1 m_1
0	0	1	0	1 m_2
0	0	1	1	0
0	1	0	0	1 m_4
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1 m_8
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$f(w, x, y, z)$
 $= \sum (m_0, m_1, m_2, m_4, m_8)$

wx	yz: 00	01	11	10
00	0 1	1 1	3	1
01	4 1	5	7	6
11	12	13	15	14
10	8 1	9	11	10

$= w'x'y' + w'y'z' + w'x'z'$
 $+ x'y'z'$



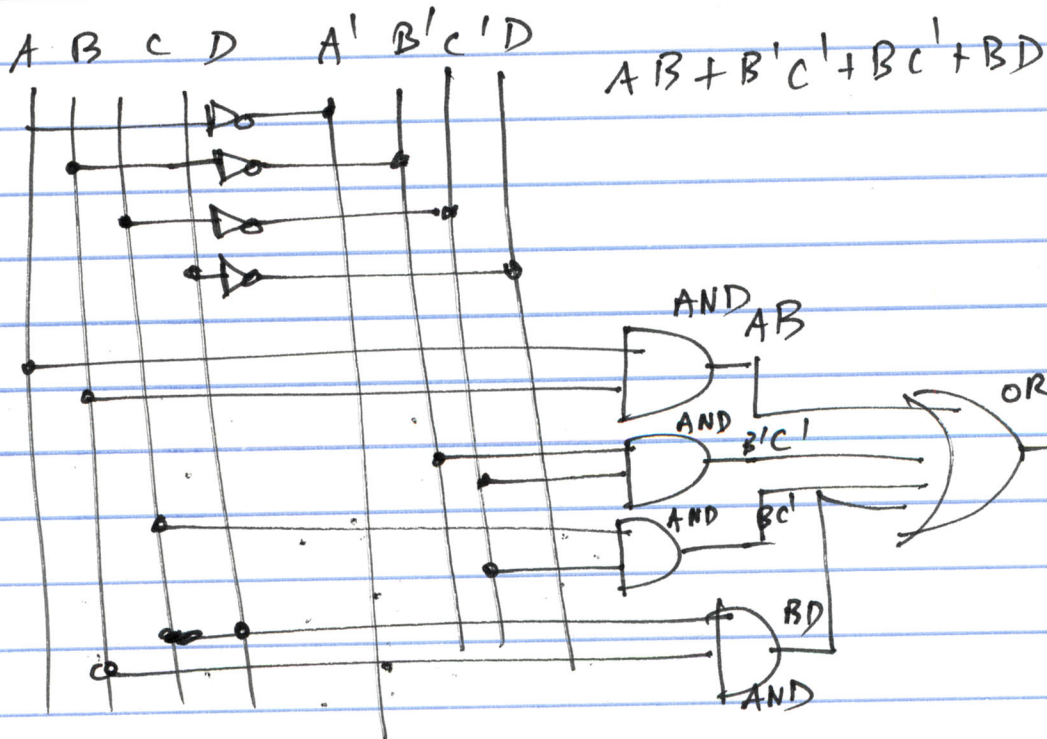
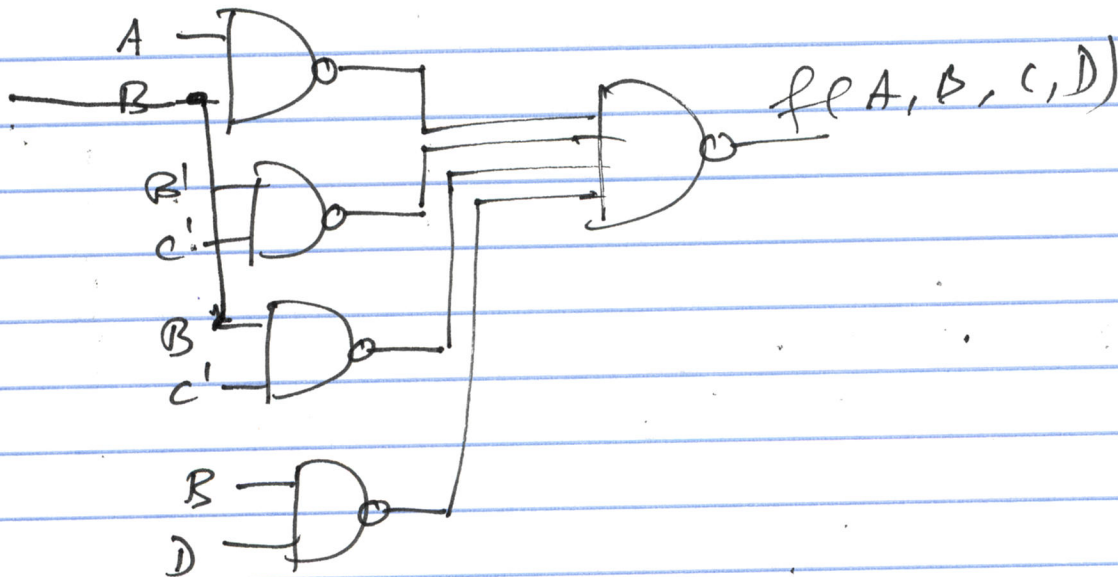
(5)

NAND only implementation March 8, 2017

(4)

$$F(A, B, C, D) = AB + B'C' + B(C' + D) \\ = AB + B'C' + Bc' + BD$$

Convert to standard "sum of products" form



5

Signed Binary Arithmetic

Step 1 convert A, B to binary; obtain $-A, -B$
2's complement

Hint
7

10001
00010001

2 66
2 33 - 0
2 16 - 1
2 8 - 0
2 4 - 0
2 2 - 0
1 - 0

A

0	1	0	0	0	0	1	0
---	---	---	---	---	---	---	---

2 72
2 36 - 0
2 18 - 0
2 9 - 0
2 4 - 1
2 2 - 0
1 - 0

B

0	1	0	0	1	0	0	0
---	---	---	---	---	---	---	---

-A 2's complement of +A

0	1	0	0	0	0	1	0		A
1	0	1	1	1	0	1	1		1's comp.
									+1
1	0	1	1	1	1	0	1		2's comp

-A

1	0	1	1	1	1	0	1
---	---	---	---	---	---	---	---

2's complement of B

1 0 1 1 0 1 1 1
+ 1

-B

1	0	1	1	1	0	0	0
---	---	---	---	---	---	---	---

a. $X = A + B$

0	1	0	0	0	0	1	0		
+	0	1	0	0	1	0	0	0	
									↓
1	0	0	0	1	0	1	0		

since carry into the MSB \neq carry out of the MSB
it is a overflow

b. $Y = A - B$

~~$Y = A - B$~~
 $= A + (-B)$
 $= A + 2's complement of B$

0	1	0	0	0	0	1	0		
+	1	0	1	1	0	0	0	0	
									↓
1	1	1	1	0	1	0	0		

no overflow
result is negative
automatically it is in
2's complement form

(1)

March 10, 2017

c. $Z = -A - B = (-A) + (-B)$
 $= 2's \text{ complement of } A + 2's \text{ complement of } B$

$$\begin{array}{r}
 0^{\text{th}} \quad 1 \quad 1 \quad 1 \\
 1011110 \\
 + 10111000 \\
 \hline
 01110110
 \end{array}$$

Carry into the MSB = 0
 Carry out of the MSB = 1
 overflow has occurred.

d. $W = -A + B = (-A) + B$
 $2's \text{ complement of } A + B$

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 1 \\
 1011110 \\
 + 01001000 \\
 \hline
 00000110
 \end{array}$$

72 - 66
 +6

no overflow \Rightarrow answer is positive.

(b) 00001000 8 bits

8 bit container

(2)

March 10, 2017

(6)

$$f(w, x, y, z) = \Sigma(0, 2, 4, 6, 10, 14)$$

$$d(w, x, y, z) = \Sigma(3, 8, 9, 11, 12, 13, 15)$$

		yz			
	wx	00	01	11	10
00		0 1	1 1	3 d	2 1
01		4 1	5	7	6 1
11		12 d	13 d	15 d	14 1
10		8 d	9 d	11 d	10 1

1 group of 8's:

$$f(w, x, y, z) = \Sigma(12)$$

z'

$$f(w, x, y, z)$$

~~(7) z z z z~~

March 10, 2017

7

$F(w, x, y, z) = \sum (0, 2, 4, 6, 10, 14)$
 Implement using a 4x1 Mux.

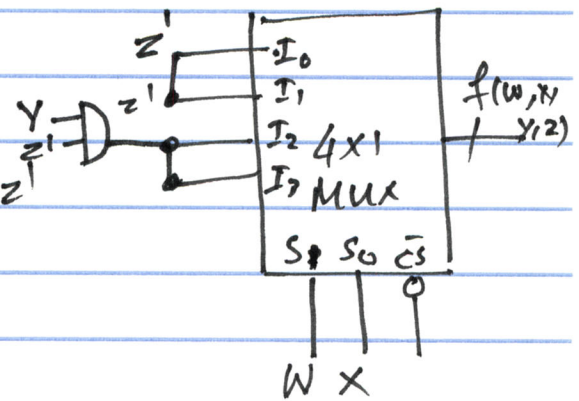
w	x	y	z	$f(w, x, y, z)$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

$I_0 = ? \quad y'z' + yz' = z'(y+y) = z'$

$y'z' + yz' = z'(y+y) = z'$

$I_2 = yz'$

$I_3 =$



7

March 10, 2017

$$F(a,b,c) = \sum (0, 2, 4, 6)$$

