Boolean algebra B. Algebrai simplification

1. literals x , y , A , B
   single x' A'

   constants { 0 , 1 } binary

   bit arithmetic

C_{in}		S	C_{out}
0	0+0+0	0	0
0	0+0+1	1	0
0	0+1+0	1	0
0	0+1+1	0	1
1	1+0+0	1	0
1	1+0+1	0	1
1	1+1+0	0	1
1	1+1+1	1	1

2. Terms Boolean algebraic terms

   need operators

   Boolean operators { + , · , ' }

   } OR AND NOT {

   f(x,y,z) x·y·z product term ✓

   (xyz)

   (x+y+z) Sum term (dual)

3. You use terms to make expressions functions are defined by expressions

$$f(x,y,z) = \underset{\uparrow \text{AND}}{x \cdot y \cdot z} + \underset{\uparrow \text{OR}}{x' \cdot y' z} \quad (\leftarrow \text{NOT}) = xyz + x'y'z$$

4. Product terms

$f(x,y)$ — 2 variable product terms

x	y	~~f(x,y)~~
0	0	$x'y'$
0	1	$x'y$
1	0	xy'
1	1	xy

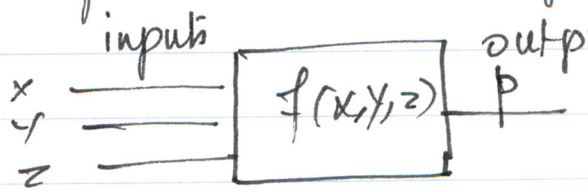
$f(x,y,z)$ Truth table : versatile tool

product terms 3 — 0, 1 — $2^3 = 8$ product terms

Minterm	x	y	z	
m_0	0	0	0	$x'y'z'$
m_1	0	0	1	$x'y'z$
m_2	0	1	0	$x'yz'$
m_3	0	1	1	$x'yz$
m_4	1	0	0	$xy'z'$
m_5	1	0	1	$xy'z$
m_6	1	1	0	xyz'
m_7	1	1	1	xyz

Complete product terms "minterms"

5. functions): A function takes n inputs, performs Boolean operations of n inputs, and produces an output. "Block diagram"



Express the function using "sum" of minterms

$f_2(x,y,z) = x'y'z + x'yz + xy'z + xy'z'$

$= \Sigma(1,3,4,5) = m_1 + m_3 + m_5 + m_4$

6. Implement the function

$$f_2(x,y,z) = x'y'z + x'yz + xy'z + xy'z'$$

AND OR NOT

We need logic gates

6a. Logic gate:

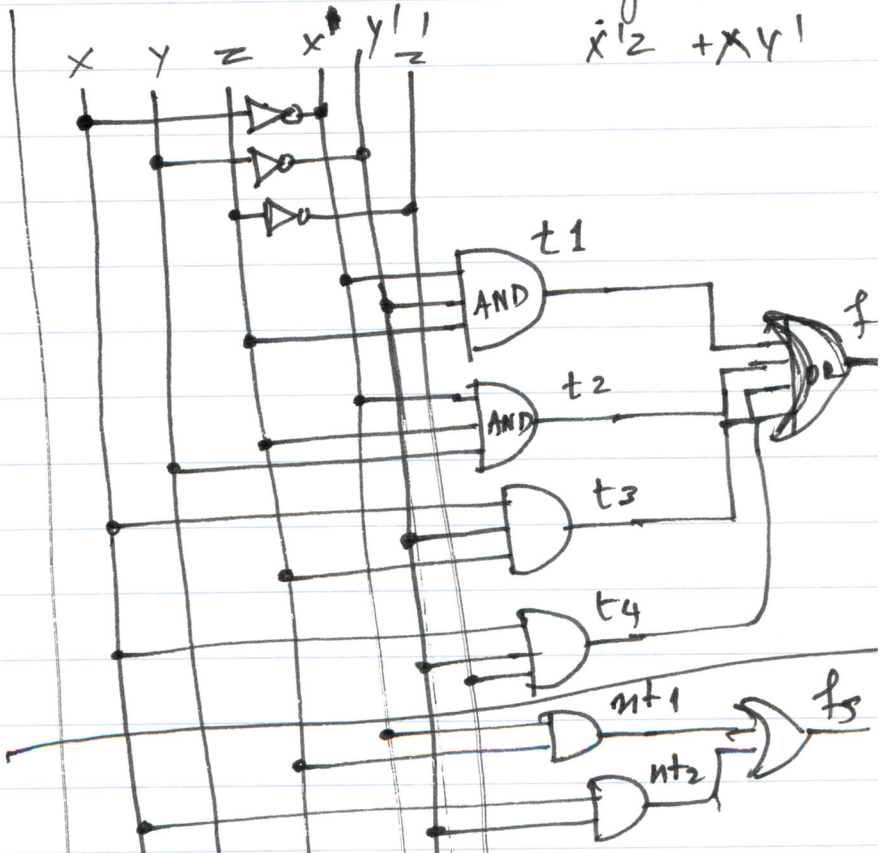
1. name of the gate
2. Graphic symbol
3. Algebraic symbol
4. algebraic expression
5. Truth table explaining its operation

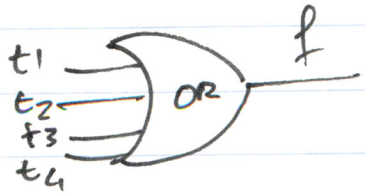
1. AND.
2. 

3. ~~x · y~~ ·
4. $x \cdot y$
5.

x	y	x.y
0	0	0
0	1	0
1	0	0
1	1	1

$x'z + xy'$





7. Simplification: Minimization

Goal of simplification:

1. ~~reduct~~ Reduce the number of terms
2. Reduce the # of literals / term $\Rightarrow$ less gates

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$f_2(x,y,z) = x'y'z + x'yz + xy'z + xy'z'$

Simplify this to minimum no. of terms of literals.

use ~~the~~ Boolean Algebra.

$x'(y'z + yz) + x(y'z + y'z')$

$= x'((y' + y)) + x(y'((z + z')))$

$(y' + y) \rightarrow 1$

$(z + z') \rightarrow 1$

$= x'(z \cdot 1) + x(y' \cdot 1)$

$= x'z + xy'$

We went from 4 terms of 3 literals each to 2 terms

each term has only 2 literals

from $4 \times 3 = 12$ literals $\Rightarrow 2 \times 2 \Rightarrow 4$ literal

$12 \rightarrow 4$

lets implement this!