Entity-Relationship Data Model

Proposed by P. Chen in 1976.

Used for the description of the *conceptual schema* of the database.

Formal notation but close to natural language.

Can be *mapped* to various data models:

- relational
- object-oriented
- XML
- ...

Textbook, chapters 6 and 7

Basic ER model concepts

Instance level	*Schema level*
Entity	Entity type
Relationship (instance)	Relationship type
	Cardinality constraints
Attribute value	Attribute
Key value	Key

Entities

Entity: something that exists and can be distinguished from other entities.

Examples: a person, an account, a course.

Entity type: a set of entities with similar properties.

Examples: persons, employees , Citibank accounts, courses.

Entity types can overlap.

Entity type extension: the set of entities of a given type in a given database instance.

Notation:

entities: $e_1, e_2, \ldots$

"entity e is of type T": $T(e)$.

Attributes

Domain: a predefined set of primitive, atomic values.

Examples: integers, character strings, decimals.

Entity types *are not* domains.

Attribute: a (partial) function from an entity type to a domain.

Attributes represent *properties* of entities.

Examples:

`Name : Person` $\longrightarrow$ `String`

`Balance : Account` $\longrightarrow$ `Decimal`

Notation:

$A(e)$: "the value of the attribute A for the entity e".

Example:

`Name(`e_1`)='Brown'`

Keys

Key: a (minimal) set of attributes that uniquely identifies every entity in an entity type.

This is a schema-level notion.

Examples:

Entity type	**Key**
Americans	SSN
ATT accounts	Phone number
NY vehicles	License plate number
US vehicles	(License plate number,State)

There may be more than one key for an entity type. One is picked as the *primary* key.

Relationships

Relationship type of arity k: a subset of the Cartesian product of some entity types $E_1, \ldots, E_k$.

Examples:

`Teaches(Employee,Class)`

`Supplies(Supplier,Customer,Product)`

`Parent(Person,Person)`

Relationship types represent associations between entity types. They can have attributes.

Relationship instance of arity k: a k-tuple of entities of the appropriate types.

Examples:

`Teaches(`e_1`,`c_1`)` where `Employee(`e_1`)` and `Class(`c_1`)` and `Name(`e_1`)='Brown'.`

Cardinality constraints

Binary relationship type $R(A, B)$ is:

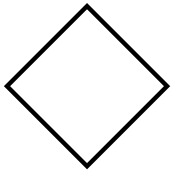
$\boxed{1:1}$ if for every entity e_1 in A there is at most one entity e_2 in B such that $R(e_1, e_2)$ and *vice versa*.

$\boxed{\text{N}:1}$ if for every entity e_1 in A there is at most one entity e_2 in B such that $R(e_1, e_2)$.

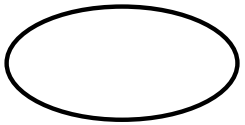
$\boxed{\text{N}:\text{M}}$ otherwise.

Diagrams I

Entity type



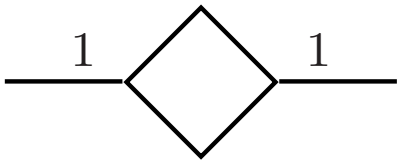
Relationship type



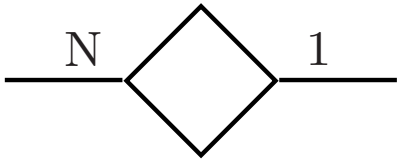
Attribute

Key attributes are underlined.

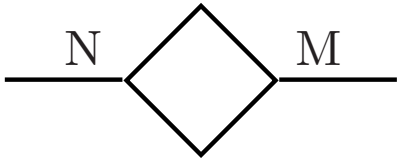
Diagrams II



1:1



N:1



N:M

Advanced schema-level concepts

- **isa** relationships.
- weak entity types.
- complex attributes
- roles.

isa relationships

A **isa** B if every entity in the entity type A is also in the entity type B.

Example: `Faculty` **isa** `Employee`.

This is a schema-level notion.

If A **isa** B, then:

$Attributes(B) \subseteq Attributes(A)$ (*inheritance* of attributes),

$Key(A) = Key(B)$ (*inheritance* of key).

Example:

$$\texttt{Rank} : \texttt{Faculty} \rightarrow \{'\text{Assistant}', '\text{Associate}', \ldots\}$$

`Rank` is not defined for non-faculty employees (or defined differently).

Weak entity types

A is a *weak* entity type if:

- it does not have a key.
- the entities in A can be identified through an identifying relationship type $R(A, B)$ with another entity type B.

The entities in A can be identified by the combination of:

- the *borrowed* key of B.
- some *partial* key of A.

Example.

Entity types: `Employee`, `Dependent`.

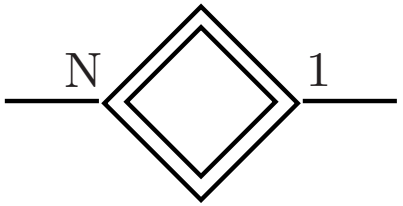
Identifying relationship type: `DepOf(Dependent,Employee)`.

Borrowed key (of `Employee`): `Name`.

Partial key (of `Dependent`): `FirstName`.

Diagrams III

isa relationship



Identifying relationship

Weak entity type

Complex attributes

Attribute values can be:

- sets (*multivalued* attributes).
- tuples (*composite* attributes).

Examples:

Multivalued attribute:

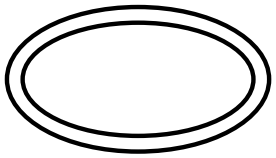
$$\texttt{Degrees} : \texttt{Faculty} \rightarrow 2^{\{'\text{B.A.}','\text{B.S.}',\ldots,'\text{Ph.D.}',\ldots\}}$$

Composite attribute:

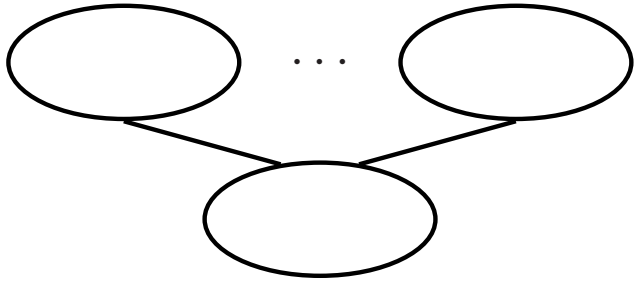
$$\texttt{Address} : \texttt{Employee} \rightarrow \texttt{Street} \times \texttt{City} \times \texttt{Zipcode}$$

Multivalued and composite attributes can be expressed using other constructs of the E-R model.

Diagrams IV



Multivalued
attribute



Composite
attribute

Roles

Roles are necessary in a relationship type that relates an entity type to itself. Different occurrences of the same entity type are distinguished by different *role names*.

Example.

In the relationship type

$$\texttt{ParentOf}(\texttt{Person}, \texttt{Person})$$

the introduction of role names gives

$$\texttt{ParentOf}(\texttt{Parent} : \texttt{Person}, \texttt{Child} : \texttt{Person})$$

ER design

General guidelines:

- schema: stable information, instance: changing information.
- avoid redundancy (each fact should be represented once).
- no need to store information that can be computed.
- keys should be as small as possible.
- introduce artificial keys only if no simple, natural keys available.

How to choose entity types:

- things that have properties of their own, or
- things that are used in navigating through the database.
- avoid null attribute values if possible by introducing extra entity types.

isa relationship design

Generalization (bottom-up):

- generalize a number of different entity types (with the same key) to a single type.
- factor out common attributes.

Example:

`Student` **isa** `Person`

`Teacher` **isa** `Person`

`Name` : `Person` → `String`

Specialization (top-down):

- specialize an entity type to one or more more specific types.
- add attributes in more specific entity types.

Example:

`Salary` : `Teacher` → `Decimal`

Mapping ER diagrams to relations

Assumption: *no complex attributes.*

Multiple stages:

1. creating relation schemas from entity types.
2. creating relation schemas from relationship types.
3. identifying keys.
4. identifying foreign keys.
5. schema optimization.

Mapping entity types to relations

Entity type	*Relation schema*
E_1 such that E_1 **isa** E_2	$Key(E_2)$ $\cup(Attrs(E_1) - Attrs(E_2))$
E_1 is a weak entity type identified by $R(E_1, E_2)$	$Key(E_2)$ $\cup(Attrs(E_1) - Attrs(E_2))$
E_1 is none of the above	$Attrs(E_1)$

Mapping relationship types to relations

Relationship type	*Relation schema*
$R(E_1, \ldots, E_n)$	$Key(E_1) \cup \cdots Key(E_n)$ $\cup Attrs(R)$

No relations are created from **isa** or identifying relationships.

Different occurrences of the same attribute name should be named differently.

Identifying keys

Relation schema W is the result of mapping an entity type E_1 or a relationship type $R(E_1, E_2)$.

Source of W	*Key of W*
Entity type E_1	$Key(E_1)$
Weak entity type E_1	Union of borrowed and partial keys of E_1
$R(E_1, E_2)$ is $1:1$	$Key(E_1)$ or $Key(E_2)$
$R(E_1, E_2)$ is $N:1$	$Key(E_1)$
$R(E_1, E_2)$ is $N:M$	$Key(E_1) \cup Key(E_2)$

These rules can be generalized to arbitrary relationship types $R(E_1, \ldots, E_n)$.

Identifying foreign keys

Relation schema W is the result of mapping an entity type E_1 or a relationship type $R(E_1, E_2)$.

Source of W	*Foreign keys of W*
Entity type E_1	No foreign keys
Weak entity type E_1	Borrowed key of E_1
Entity type E_1 such that E_1 **isa** E_2	$Key(E_1)$
$R(E_1, E_2)$	$Key(E_1)$, $Key(E_2)$

Schema optimization

Combine relation schemas with *identical* keys coming from the same entity type.

Example.

COURSE

CNAME	*CNUMBER*

and

MEETS_IN

CNUMBER	*BUILDING*	*RNUMBER*

can be combined to yield:

COURSE

CNAME	*CNUMBER*	*BUILDING*	*RNUMBER*

Designing relational databases

Method 1:

1. design an E-R schema.
2. map it to a relational schema.

Method 2:

1. design an E-R schema.
2. map it to a relational schema.
3. modify the relational schema.

Method 3:

1. start with a single relational schema containing all attributes.
2. decompose it.

The schema resulting from Method 1 is guaranteed to be "good", while those resulting from Methods 2 and 3 have to be analyzed for "goodness".

"Good" and "bad" database schemas

"Bad" schema:

- **Repetition** of information. Leads to **redundancy** and update **anomalies**.
- **Inability to represent** information. Leads to **anomalies** in insertion and deletion.

"Good" schema:

- relation schemas in **normal form** (anomaly-free): 3NF, BCNF.
- decompositions have the lossless join property and preserve dependencies.

Functional dependencies (FDs)

Relation schema $R(A_1, \ldots, A_n)$.

Sets of attributes: $X, Y, Z, \ldots \subseteq \{A_1, \ldots, A_n\}$.

Functional dependency: a pair $X \to Y$.

Notation:

- $A_1 \cdots A_n$ instead of $\{A_1, \ldots, A_n\}$.
- XY instead of $X \cup Y$.

Theory of functional dependencies

FD satisfaction: r satisfies $X \to Y$ if for all tuples $t_1, t_2 \in r$:

if $t_1[X] = t_2[X]$, then also $t_1[Y] = t_2[Y]$.

Entailment: A set of FDs F entails $X \to Y$, if every relation that satisfies all the dependencies in F, also satisfies $X \to Y$.

Notation: $F \models X \to Y$ (F entails $X \to Y$).

Closure of a dependency set F: the set of dependencies entailed by F.

Notation: $F^+ = \{X \to Y : F \models X \to Y\}$.

Keys

Given $R(A_1, \ldots, A_n)$ and a set of dependencies F over R.

$X \subseteq \{A_1, \ldots, A_n\}$ is a **key** of R if:

1. the dependency $X \rightarrow A_1 \cdots A_n$ is in F^+.
2. for all proper subsets Y of X, the dependency $Y \rightarrow A_1 \cdots A_n$ is not in F^+.

Related notions:

- *primary key:* one designated key.
- *candidate key:* one of the keys.
- *superkey:* superset of a key.

Inference of functional dependencies

Problem: how to tell whether $X \to Y \in F^+$.

Notation: U - the set of all the attributes of R.

Inference rules (Armstrong axioms):

1. *reflexivity:* if $Y \subseteq X \subseteq U$, then infer $X \to Y$ (*trivial* dependency).
2. *augmentation:* if $Z \subseteq U$, then from $X \to Y$ infer $XZ \to YZ$.
3. *transitivity:* from $X \to Y$ and $Y \to Z$, infer $X \to Z$.

Properties of axioms

Armstrong axioms are:

- **sound:** if $X \rightarrow Y$ is derived from F, then $X \rightarrow Y \in F^+$.
- **complete:** if $X \rightarrow Y \in F^+$, then $X \rightarrow Y$ is derived from F.

Additional (implied) inference rules:

4. **union:** from $X \rightarrow Y$ and $X \rightarrow Z$, infer $X \rightarrow YZ$.
5. **decomposition:** if $Z \subseteq Y$, then from $X \rightarrow Y$ infer $X \rightarrow Z$.

Boyce-Codd Normal Form (BCNF)

Notation:

- R is a relation schema.
- F is the set of FDs associated with R.
- F^+ is the dependency closure of F.
- A is an attribute of R.

R is in BCNF if for every functional dependency $X \rightarrow A \in F^+$:

- $A \in X$ (trivial FD), or
- X contains a key of R.

Each instance of a relation schema which is in BCNF does not contain a redundancy (that can be detected using FDs alone).

Third Normal Form (3NF)

R is in 3NF if for every functional dependency $X \rightarrow A \in F^+$:

- $A \in X$ (trivial FD), or
- X contains a key of R, or
- A is part of some key of R.

If R is in BCNF, it is also in 3NF.

There are relations that are in 3NF but not in BCNF:

CSZ

CITY	STREET	ZIP

with dependencies:

$$\text{CITY STREET} \rightarrow \text{ZIP}$$

$$\text{ZIP} \rightarrow \text{CITY}$$

Redundancies in 3NF

A relation schema in 3NF may still have an instance with redundancies.

Example.

CSZ

CITY	STREET	ZIP
SF	First	77077
SF	Third	77077

The dependency ZIP $\rightarrow$ CITY violates BCNF and identifies a redundancy.

Decompositions

Decomposition: replacement of a relation schema R by two relation schema R_1 and R_2 such that:

- both R_1 and R_2 are subsets of R,
- $R_1 \cup R_2 = R$.

Lossless decomposition: (R_1, R_2) is a lossless decomposition of R with respect to a set of FDs F if for every instance r of R that satisfies F:

$$\pi_{R_1}(r) \bowtie \pi_{R_2}(r) = r.$$

A simple criterion for checking whether a decomposition (R_1, R_2) is lossless:

- $F \models R_1 \cap R_2 \rightarrow R_1$, or
- $F \models R_1 \cap R_2 \rightarrow R_2$.

A sequence of decompositions of R into R_1 and R_2, R_1 into R_1' and R_1'' etc. may be viewed as a decomposition of R into more than two relation schemas.

Dependency preservation

Dependencies associated with relation schema R_1 and R_2 in a decomposition (R_1, R_2):

$$F_{R_1} = \{X \to Y | X \to Y \in F^+ \wedge XY \subseteq R_1\}$$

$$F_{R_2} = \{X \to Y | X \to Y \in F^+ \wedge XY \subseteq R_2\}.$$

(R_1, R_2) preserves a dependency f iff $f \in (F_{R_1} \cup F_{R_2})^+$.

Decomposition into BCNF

Notation:

- F - set of FDs associated with R.

ALGORITHM:

For some nontrivial nonkey dependency $X \rightarrow A$ in F^+:

1. create a relation with the schema XA.
2. remove A from R.

If the resulting schemas are not in BCNF, decompose them further.

This algorithm produces a lossless decomposition into BCNF which does not have to preserve dependencies.

Decomposition (synthesis) into 3NF

Minimal cover F' for F:

- set of FDs equivalent to F ($F^+ = (F')^+$),
- all FDs in F' are of the form $X \to A$,
- further simplification by removing dependencies or attributes from dependencies in F' yields a set of FDs inequivalent to F.

ALGORITHM:

Create F'.

Create a relation XA for every dependency $X \to A \in F'$.

Create a relation X for some key X of R.

Remove redundancies.

This algorithm produces a lossless decomposition into 3NF which preserves dependencies.

Multivalued dependencies (MVDs)

Relation schema $R(A_1, \ldots, A_n)$.

Sets of attributes: $X, Y, Z, \ldots \subseteq \{A_1, \ldots, A_n\}$.

Multivalued dependency: a pair $X \rightarrow\!\!\!\rightarrow Y$.

MVD satisfaction: r satisfies $X \rightarrow\!\!\!\rightarrow Y$ if for all tuples $t_1, t_2 \in r$:

> if $t_1[X] = t_2[X]$, then there is a tuple $t_3 \in r$ such that
> $t_3[XY] = t_1[XY]$ and $t_3[Z] = t_2[Z]$,

where $Z = \{A_1, \ldots, A_n\} - XY$.

Entailment: defined in the same way as for FDs.

Fourth Normal Form (4NF)

F is the set of FDs and MVDs associated with a relation schema
$R = \{A_1, \ldots, A_n\}$.

R is in 4NF if for every multivalued dependency $X \rightarrow\rightarrow Y$ entailed by F:

- $Y \subseteq X$ or $XY = \{A_1, \ldots, A_n\}$ (trivial MVD), or
- X contains a key of R.