Data Integration: Datalog

Jan Chomicki

University at Buffalo and Warsaw University

Feb. 22, 2007

Plan of the course

1. Datalog
2. Negation
3. Schema mapping
4. Data integration and exchange
5. Schematic discrepancies
6. Query evaluation
7. Inconsistency and incompleteness
8. XML databases
9. XML query languages
10. XML data integration
11. Semantic Web

Datalog [AHV95]

A logical language:

- Datalog programs consist of logical facts and rules
- Datalog is a subset of Prolog (no data structures).

Basic concepts:

- term: constant, variable
- predicate (relation)
- atom
- clause, rule, fact
- substitution
- unification

Logic programs

Atom

- syntax: $P(T_1, \ldots, T_n)$
- semantics: predicate P is true of terms T_1 and ... and T_n

Implication (clause)

- syntax: $A_0 \ :- \ A_1, \ldots, A_k$
- semantics: atom A_0 is true if atoms A_1 and $\cdots$ and A_k are true
- $k = 0$: fact (ground if no variables)
- $k > 0$: rule consisting of head A_0 and body $A_1, \ldots, A_k$
- all the variables universally quantified
- all the variables in the head occur also in the body

Logic program P

- $EDB(P)$: a set of ground facts encoding a database instance
- $IDB(P)$: a set of rules encoding a query, with a special predicate `query` to return the result

Example logic program

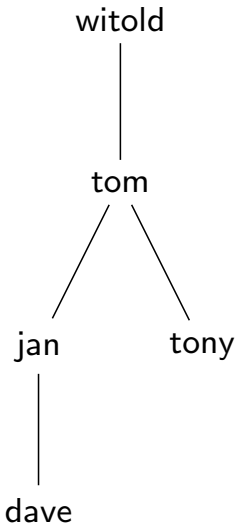
Ancestry

```
%% Facts
parent(witold,tom).
parent(tom,jan).
parent(tom,tony).
parent(jan,dave).

%% Rules
anc(X,Y) :- parent(X,Y).
anc(X,Z) :- parent(X,Y), anc(Y,Z).

%% Query 1
query(X) :- anc(X,dave).

%% Query 2
query(X) :- anc(X,dave), anc(X,tony).
```



Logical semantics

The least Herbrand model M_P of a program P

- Herbrand: constants interpreted as themselves
- Least: contained in all the Herbrand models of P

Properties of M_P

- M_P: a subset of the Herbrand base (the set of all possible ground atoms) of P
- M_P = set of ground facts implied by P

Query answer

A tuple t is an answer to the query Q encoded by $IDB(P)$ in the database encoded by $EDB(P)$ if $\texttt{query}(t) \in M_P$.

Ancestry

M_P consists of all parent facts and:

```
%% First-level ancestors
anc(witold,tom).
anc(tom,jan).
anc(tom,tony).
anc(jan,dave).

%% Second-level ancestors
anc(witold,jan).
anc(witold,tony).
anc(tom,dave).

%% Third-level ancestors
anc(witold,dave).
```

Substitutions

Substitution

- mapping from variables to terms
- ground: all the terms in the range are constants

Substitutions can be applied to atoms and clauses:

- instance $h(A)$ is obtained by replacing all the occurrences of variable x in A by the term $h(x)$ (for every variable)

Substitutions can be composed:

$$(h \circ g)(x) = g(h(x)).$$

Ground program $ground(P)$

All the ground instances of the clauses in P, constructed using the constants in P (active domain).

Fixpoint semantics

Given:

- a logic program P
- a set of ground facts I.

T_P operator

$$T_P(I) = \{A \mid \exists r \in ground(P).\ r = A :- A_1, \ldots, A_n \wedge A_1 \in I \wedge \cdots A_n \in I.\}$$

Fixpoint

I is a fixpoint of T_P if $T_P(I) = I$.

Properties of T_P

1. T_P is monotonic, i.e.. $I \subseteq J \Rightarrow T_P(I) \subseteq T_P(J)$
2. T_P has a least fixpoint $lfp(T_P)$ contained in all the fixpoints
3. $lfp(T_P) = M_P$
4. $lfp(T_P)$ is obtained by iterating T_P finitely many times

Bottom-up query evaluation

Naive evaluation

1. multiple iterations: compute $T_P(\emptyset), T_P(T_P(\emptyset)), \ldots$ until result does not change
2. evaluation terminates: result is M_P
3. retrieve query predicate facts from M_P

!!! facts rederived multiple times

Semi-naive evaluation

- the result is accumulated
- at least one new fact is used in each rule application
- rule r has with only EDB atoms in the body: r is applied only in the first iteration
- rule r has IDB atoms $A_1, \ldots, A_n$ in the body: in every iteration r is applied n times, each time considering for A_i only the facts added in the last iteration
- further optimizations are possible

Top-down evaluation

Ground case

Evaluation of a ground atom A:

1. A succeeds immediately if there is a fact A in the program P
2. A succeeds if there is a clause $A :- A_1, \ldots, A_n \in ground(P)$ such that $A_1, \ldots, A_n$ succeed

General case

Evaluation of a non-ground atom A:

1. unify A with a fact or a rule head (after renaming apart) in P
2. propagate the substitutions to the body of the rule
3. evaluate the body

Unification

Most general unifier

A substitution h:

- unifies two atoms A and B if $h(A) = h(B)$.
- is a most general unifier (mgu) of A and B if for all other substitutions g unifying A and B, there is a substitution g' such that $g = h \circ g'$.

Unification algorithm

Returns failure or an mgu h of A and B, given two atoms $A = P(x_1, \ldots, x_n)$ and $B = Q(y_1, \ldots, y_k)$.
If $P \neq Q$ or $n \neq k$: return failure. Otherwise $Pairs = (x_i, y_i) \mid i = 1, \ldots, n\}$ and $Pairs^*$ is the reflexive symmetric transitive closure of $Pairs$:

1. If any equivalence class of $Pairs^*$ contains two different constants: return failure.
2. Otherwise for every equivalence class of $Pairs^*$:
   1. if the class contains a constant c, then for every variable x in the class, $h(x) = c$,
   2. otherwise, pick a variable x_0, which is the smallest in some fixed ordering of variables, in the class, and for every other variable x in the class, $h(x) = x_0$.

General logic programs

Syntax

- terms can be built using function symbols: $f(0), f(f(X)), \ldots$
- terms encode data structures

Semantics

- active domain contains ground terms
- least Herbrand model M_P = least fixpoint $lfp(T_P)$
- M_P can be infinite

Query evaluation

- bottom-up or top-down: may fail to terminate
- an undecidable problem
- unification: requires occur check


S. Abiteboul, R. Hull, and V. Vianu.
Foundations of Databases.
Addison-Wesley, 1995.