Temporal Data Exchange

Ladan Golshanara[1], Jan Chomicki[1], and Wang-Chiew Tan[2]

1 **State University of New York at Buffalo, NY, USA**
ladangol@buffalo.edu, chomicki@buffalo.edu
2 **Recruit Institute of Technology and UC Santa Cruz, CA, USA**
tan@cs.ucsc.edu

Abstract

Data exchange is the problem of transforming data that is structured under the source schema into data structured under another schema, called the target schema, so that both source and target data satisfy the relationship between the schemas. Many applications such as planning, scheduling, medical and fraud detection systems, require data exchange in the context of temporal data. Even though the formal framework of data exchange for relational database systems is well-established, it does not immediately carry over to the setting of temporal data, which necessitates reasoning over unbounded periods of time.

In this work, we study data exchange for temporal data. We first motivate the need for two views of temporal data: the concrete view, which depicts how temporal data is compactly represented and on which implementations are based, and the abstract view, which defines the semantics of temporal data. We show how the framework of data exchange can be systematically extended to temporal data. The core of our framework consists of two new chase algorithms: the abstract chase over an abstract temporal instance and the concrete chase over a concrete temporal instance. We show that although the two chase procedures operate over fundamentally different views of temporal data, the result of the concrete chase is semantically aligned with the result of the abstract chase. To obtain the semantic alignment, the nulls (which are introduced by data exchange and model incompleteness) in both the concrete view and the abstract view are annotated with temporal information. Furthermore, we show that the result of the concrete chase provides a foundation for query answering. We define naïve evaluation on the result of the concrete chase and show it produces certain answers.

Keywords and phrases Data Exchange, Temporal Database, Chase, Incomplete information, Abstract view, Concrete view

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1 Introduction

Temporal data refers to historical data or data that is dated. Temporal data is needed by many organizations and individuals to support audit trails. With temporal data one can represent when a fact is true and for how long [6]. The temporality of facts is also critical in diverse domains, from medical diagnosis to assessing the changing business conditions of companies [17]. In fact, temporal database features, which were recently added to the SQL:2011 standard [12], have already been adopted by major database management systems such as DB2, Oracle, and Teradata.

Temporal database is a relational database where there is at least one temporal attribute associated with each relation schema. The need to store information that span over long periods of time necessitates that temporal attribute values be intervals so that multiple (sometimes infinitely many) time points can be compactly represented. For example, the fact that Ada worked in IBM between 2010 and 2013 is usually represented as (Ada, IBM,

[2010, 2014)) where [2010, 2014) is a *clopen interval* that denotes the years 2010, 2011, 2012, and 2013. The fact that Ada has worked in Intel since then can be represented as (Ada, Intel, [2014, ∞)). An infinite interval, such as [2014, ∞), is a useful abstraction when the endpoint is not provided. However, it is simpler and more natural to view information at every snapshot, i.e. at discrete time points. For these reasons, prior work on temporal databases [5, 10, 19] has provided two views of temporal data: the *concrete temporal view* (or *concrete view* in short) and the *abstract temporal view* (or *abstract view* in short). In the concrete view the same data over multiple time points is compactly stored using time intervals, such as the examples with Ada above. This view is an extension of the relational model where the temporal attribute takes clopen intervals as values. On the other hand, in the *abstract temporal view* (or *abstract view* in short) there is a fact representing the data at every time point when it is true. For example, there are four abstract tuples (Ada, IBM, 2010), (Ada, IBM, 2011), (Ada, IBM, 2012), (Ada, IBM, 2013) corresponding to the concrete tuple (Ada, IBM, [2010, 2014)), and infinitely many abstract tuples (Ada, Intel, 2014), (Ada, Intel, 2015), ... that correspond to the concrete tuple (Ada, Intel, [2014, ∞)). The abstract view is based on the relational model with infinite relations. The semantic alignment between concrete and abstract views is formally defined as the *semantic mapping* in Section 3. Note that SQL:2011 [12] implicitly supports both views. It provides features for defining tables with interval endpoint attributes, while the semantics of integrity constraints is defined with respect to time points.

Data exchange [7] refers to the problem of translating data that conforms to one schema (called the *source schema* R_S) into data that conforms to another schema (called the *target schema* R_T) given a specification of the relationship between the two schemas. This relationship is specified by means of a *schema mapping* consisting of a set of *source-to-target tuple generating dependencies (s-t tgds)* and a set of *tuple generating dependencies (tgds)* and *equality generating dependencies (egds)* on the target schema. Given a schema mapping and a source instance I, the goal of data exchange is to materialize a target instance J that satisfies the specification (i.e. (I, J) satisfies s-t tgds and J satisfies tgds and egds). Such an instance J is called a *solution* for I w.r.t the given schema mapping. For a given source instance, there may be no solution since there may not exist a target instance that satisfies the specification. On the other hand, there may be many solutions. It was shown in [7] that among all solutions of a given source instance, the *universal solutions* are the preferred solutions because they are the most general. In [7], the *chase procedure* is used to find a universal solution. Universal solutions can be used to determine *certain answers* to the unions of conjunctive queries posed over a target schema. Certain answers to a query Q consist of all tuples that will be in the answer of Q in any arbitrary solution for a source instance w.r.t a schema mapping.

In this paper, we study the challenges that arise when we consider temporal data in the framework of data exchange. We examine the most basic case where we assume that every relation in a source instance has exactly one temporal attribute and source instances are complete (i.e. have no unknown values). In our framework, the *temporal schema mapping* $\mathcal{M}$ is a quadruple $\mathcal{M} = (R_S, R_T, \Sigma_{st}, \Sigma_k)$, where Σ_{st} is a set of *source-to-target temporal tuple generating dependencies* (s-t ttgds) between the source and target schema, and Σ_k is a set of *temporal key constraints* (tkcs) [4] over the target schema. Temporal key constraints are fundamental for temporal applications. We introduce two chase algorithms: *abstract chase* for the abstract view and *concrete chase* for the concrete view. The chase procedure in [7] is a sequential process and assumes finite relations. This assumption holds for the concrete view (modulo modifications to the chase procedure to manage time intervals) but not for

the abstract view which might contain infinite relations. To cope with infinite abstract relations, chase steps are applied in parallel in the abstract chase algorithm. Both proposed chase algorithms have two rounds: An *s-t ttgd round* followed by a *tkc round*. The s-t ttgd round aggregates all *s-t ttgd chase steps* that can be applied on its input while the tkc round aggregates all *tkc chase steps*. If the tkc round is successful, the result of the abstract chase algorithm (resp. concrete chase algorithm) satisfies the given temporal schema mapping $\mathcal{M}$. Otherwise, both chase algorithms fail. Note that s-t ttgd and tkc chase steps are extensions of the chase steps in [7] for temporal databases.

The concrete chase handles clopen intervals by *normalizing* the source instance. Intuitively, *normalization* makes the clopen intervals behave as time points. Normalization was previously used in the context of query answering in temporal databases [13, 20] on tuples with the same schema that agree on non-temporal attribute values. The proposed normalization operation in this paper does not have these restrictions. As a result of data exchange, unknown values may occur in target instances. More precisely, at each s-t ttgd chase step in the abstract chase and concrete chase algorithms fresh nulls are generated. As mentioned before, a concrete fact is a compact representation of a set of abstract facts; therefore, a fresh null generated in a concrete view should represent a set of distinct fresh nulls in the corresponding abstract view. In order to keep the correspondence between the concrete and abstract view of temporal data in the presence of unknown values generated by data exchange, the nulls in a concrete target instance are annotated with the clopen interval of the tuple they occur in, e.g. $N^{[s,e)}$ is an *interval-annotated null* in a concrete tuple, while the nulls in an abstract target instance are annotated with the time point of the tuple they occur in, e.g. N^{t_0} is a *point-annotated null* in an abstract tuple.

For example, consider the relational schema Emp(Name, Position, Company, Year). The concrete tuple (Ada, $N^{[2008,\infty)}$, IBM, [2008,∞)) contains an annotated null $N^{[2008,\infty)}$ and the tuple denotes that the position of Ada at IBM is unknown and the position is possibly different every year since 2008. This tuple corresponds to the abstract tuples $(Ada, N^{2008}, IBM, 2008), (Ada, N^{2009}, IBM, 2009), ...$ Labeled nulls which are used in relational data exchange to model incompleteness are not sufficient to show the connection between the concrete and abstract views after data exchange. This is shown in the proof of Theorems 7 and 8.

▶ **Example 1.** Suppose we have the following relations in the source schema that represents the employment histories of persons: $Employee1(Name, Company, Time)$ and $Employee2(Name, Position, Dept, Time)$.

A concrete source instance is shown in Figure 1, while the corresponding abstract instance is shown in Figure 2. Only the last two digits of years are shown. We want to move the data to a bigger schema with two relations: $Emp(Name, Position, Company, Time)$, and $Sal(Name, Position, Salary, Time)$.
Suppose we have the following s-t ttgds:

$$\forall n \forall c \forall t \; Employee1(n, c, t) \rightarrow \exists p \exists s \; Emp(n, p, c, t) \wedge Sal(n, p, s, t)$$

$$\forall n \forall p \forall d \forall t \; Employee2(n, p, d, t) \rightarrow \exists c \exists s \; Emp(n, p, c, t) \wedge Sal(n, p, s, t)$$

and the tkcs:

$$\forall n \forall p_1 \forall p_2 \forall c_1 \forall c_2 \forall t \; Emp(n, p_1, c_1, t) \wedge Emp(n, p_2, c_2, t) \rightarrow p_1 = p_2 \wedge c_1 = c_2$$

$$\forall n \forall p_1 \forall p_2 \forall s_1 \forall s_2 \forall t \; Sal(n, p_1, s_1, t) \wedge Sal(n, p_2, s_2, t) \rightarrow p_1 = p_2 \wedge s_1 = s_2$$

Employee1

Name	Company	Time
Ada	IBM	[08, 11)
Ada	Intel	[11, 13)

Employee2

Name	Position	Dept	Time
Ada	Developer	Computer	[08, 10)
Ada	DBA	Computer	[10, 11)

Figure 1 Concrete source instance

Employee1

Name	Company	Time
Ada	IBM	08
Ada	IBM	09
Ada	IBM	10
Ada	Intel	11
Ada	Intel	12

Employee2

Name	Position	Dept	Time
Ada	Developer	Computer	08
Ada	Developer	Computer	09
Ada	DBA	Computer	10

Figure 2 Abstract source instance

Figure 3 and Figure 5 respectively show the solutions for the concrete and abstract source instances w.r.t the above temporal schema mapping . These solutions can be obtained using the concrete and abstract chase algorithms, respectively. Note that both solutions satisfy the given temporal schema mapping. Moreover, if two annotated nulls are syntactically the same in an abstract or a concrete instance (e.g. $N^{[11,13)}$ in tables *Emp* and *Sal* in Figure 3 or J^{11} and K^{12} in Figure 5), they are generated by applying an s-t ttgd with an existentially quantified variable shared between the atoms on its right hand side. Figure 4 is the abstract view of the concrete solution in Figure 3. Observe that Figure 4 and Figure 5 are the same modulo renaming the point-annotated nulls. This is formally proved in Section 6. Specifically, corresponding point-annotated nulls in Figure 4 and Figure 5 carry the same time points (i.e. their temporal attribute value).

Note that even when the source instance is concrete, the s-t ttgds and tkcs are viewed in the abstract way; that is, by interpreting the abstract view of the source instance and applying the standard semantics of s-t ttgds and tkcs. However, for practical purposes, we store the target instance in its concrete representation since this representation is generally more compact than its abstract counterpart.

Contributions Our main contribution of this paper is the formalization and study of the framework of data exchange on temporal data. We define two chase algorithms, one for each view (abstract vs. concrete) of temporal data: the *abstract chase algorithm* and the *concrete chase algorithm*. We show the correctness of the concrete chase with respect to the abstract chase. This result is important because it enables one to implement data exchange on concrete temporal data with semantics corresponding to the abstract case. We also show that the result of the abstract chase over an abstract source instance is a universal solution and, therefore, the corresponding concrete solution provides a foundation for answering unions

Emp

Name	Position	Company	Time
Ada	Developer	IBM	[08, 10)
Ada	DBA	IBM	[10, 11)
Ada	$N^{[11,13)}$	Intel	[11,13)

Sal

Name	Position	Salary	Time
Ada	Developer	$M^{[08,10)}$	[08, 10)
Ada	DBA	$U^{[10,11)}$	[10, 11)
Ada	$N^{[11,13)}$	$V^{[11,13)}$	[11,13)

Figure 3 A concrete solution obtained by concrete chase algorithm

Emp

Name	Position	Company	Time
Ada	Developer	IBM	08
Ada	Developer	IBM	09
Ada	DBA	IBM	10
Ada	N^{11}	Intel	11
Ada	N^{12}	Intel	12

Sal

Name	Position	Salary	Time
Ada	Developer	M^{08}	08
Ada	Developer	M^{09}	09
Ada	DBA	U^{10}	10
Ada	N^{11}	V^{11}	11
Ada	N^{12}	V^{12}	12

Figure 4 Abstract view of the concrete solution shown in Figure 3

Emp

Name	Position	Company	Time
Ada	Developer	IBM	08
Ada	Developer	IBM	09
Ada	DBA	IBM	10
Ada	J^{11}	Intel	11
Ada	K^{12}	Intel	12

Sal

Name	Position	Salary	Time
Ada	Developer	M^{08}	08
Ada	Developer	O^{09}	09
Ada	DBA	P^{10}	10
Ada	J^{11}	X^{11}	11
Ada	K^{12}	Y^{12}	12

Figure 5 An abstract solution obtained by abstract chase algorithm

of conjunctive queries. Finally, we define naïve evaluation on the abstract and concrete instances obtained by the chase algorithms and show it produces certain answers.

Additional definitions and the proofs of all the theorems and lemmas in this paper are provided in the Appendix.

2 Related Work

Here we overview the previous relevant work on data exchange and temporal databases. The formal foundations of data exchange were developed by Fagin et al. in [7]. The authors showed that the chase algorithm, previously used for checking implication of data dependencies, can be used to produce a universal solution for instances of the data exchange problem. Universal solutions map homomorphically to other solutions for the source instance. This property makes them the preferred solutions to query. Universal solutions and, in general, solutions of instances of data exchange problem can contain incomplete information. Representing incomplete information and evaluating queries over them are more complex than in the complete case [1, 9]. The gap between the theoretical work on incomplete information and what has been used in practice is discussed in [8, 15]. The chase algorithm proposed in [7] produces labeled nulls for incomplete values. Relations with labeled nulls in them are called *naïve tables* [1, 9]. In data exchange the semantics of answering queries are defined in terms of *certain answers* [3, 7]. Certain answers [9] are tuples that belong to the answer of the posed query no matter which solution is used. It is shown in [7] that whenever a universal solution can be computed in polynomial time (for the class of dependencies identified in [7]) , certain answers to unions of conjunctive queries can also be computed in polynomial time. Computing certain answers for queries that have inequalities, however, is a coNP-complete problem [7]. Data exchange and incomplete information and other possible semantics for query answering are discussed in detail in [14].

The original chase, which we call the standard chase throughout the paper, assumes finite relational instances and is a sequential process. Further extensions to the standard chase were surveyed in [16]. In [2] a chase step, called *core chase step*, was proposed that consists of applying all tgds in parallel; however, the details in the case of egds were missing.

Motivated by [2], the proposed chase algorithms in this paper aggregate chase steps in two rounds to cope with possibly infinite abstract relations. The work is different from previous work on data exchange in two aspects: the abstract view requires infinite sets of tuples and the concrete view is an extension of relational model with intervals as the temporal attribute values.

The formal foundations of temporal data models and query languages were studied in [4, 5]. Abstract versus concrete temporal views were first developed in the context of the semantics of temporal query languages [19]. These two level views of temporal data later were used in program debugging and dynamic program analysis [13]. The papers [4, 5] did not discuss incomplete temporal information and its possible semantics. Koubarakis proposed a unified framework for both finite and infinite, definite and indefinite temporal data [11, 10]. His suggested framework extends *conditional tables* (a.k.a. c-tables) [9] and can be used to store facts such as roomA is booked from 2 to *sometime between 5 to 8*. He used global conditions to define the constraints on the start point or end point of a time interval. In his framework, the temporal attribute values can be unknown. C-tables are a generalization of naïve tables where a table is associated with global and local conditions specified by logical formulas. In temporal data exchange, the proposed chase algorithms produce point-annotated and interval-annotated nulls for unknown values. Since in our framework, the value of temporal attribute is known in source instances and there is no condition on the temporal attribute or non-temporal attributes as a result of data exchange, naïve tables are sufficient for representing incomplete information.

3 Preliminaries and Definitions

Time points and time intervals. Let $\mathsf{T_P} = \mathbb{N} \cup \{\infty\}$, where $\mathbb{N}$ denotes the set of natural numbers. Let $\mathsf{T_I} = \{[s, e) \mid s, e \in \mathsf{T_P}, s < e\}$. As described in the Introduction, a *clopen interval* $[s, e)$ denotes the set $\{s, s+1, ..., e-1\}$ of time points. A clopen interval is equivalent to two temporal attributes for representing the beginning and the end of the time interval [4]. This construction is also used in the SQL:2011 standard [12].

Domains. There are three domains Const, PNull, and INull. The domain Const is a set of infinitely many uninterpreted constants. The domain PNull is an infinite set of *point-annotated nulls*. A point-annotated null is a pair N^s where N is a label and s is a time point which shows the *temporal context* of N, i.e. the time point at which N occurs. Similarly, the domain INull is an infinite set of *interval-annotated nulls*. An interval-annotated null is a pair $N^{[s,e)}$ where N is a label and $[s, e)$ is a clopen interval which denotes the temporal context of N. Throughout this paper, we use *annotated nulls* to refer to both point-annotated nulls and interval-annotated nulls.

Two annotated nulls are equal if they are syntactically identical. We assume that the sets PNull, INull, Const, $\mathsf{T_P}$ and $\mathsf{T_I}$ are countably infinite and mutually disjoint.

Abstract and concrete schema and instances, semantic mapping. A *schema* is a finite sequence $\mathcal{R} =< R_1, ..., R_k >$ of relation symbols. Each *relation symbol* R_i is associated with a set $\{A_1, ..., A_{k_i}, T\}$ of attributes where the domain of A_j $(1 \leq j \leq k_i)$ is Const and the domain of T is either $\mathsf{T_P}$ or $\mathsf{T_I}$. An *abstract instance* I_a over the schema $\mathcal{R}$ is a function that associates to each relation symbol R_i an abstract relation I^{R_i}, where I^{R_i} is a subset of $(\mathsf{Const} \cup \mathsf{PNull})^{k_i} \times \mathsf{T_P}$. In other words, except for the temporal attribute, annotated nulls can occur among the values of other attributes in the tuples. A *complete abstract instance* I is a special case, where I^{R_i} is a subset of $\mathsf{Const}^{k_i} \times \mathsf{T_P}$.

Similarly, a *concrete instance* I_c over the schema $\mathcal{R}$ is a function that associates to each

relation symbol R_i a relation I^{R_i}, where I^{R_i} is a subset of $(\mathsf{Const} \cup \mathsf{INull})^{k_i} \times \mathsf{T_I}$. Likewise, a *complete concrete instance* is a special case of a concrete instance if each I^{R_i} is a subset of $\mathsf{Const}^{k_i} \times \mathsf{T_I}$. We will also sometimes write the tuple $(\bar{a}, t)$ in relation R_i as the *fact* $R_i(\bar{a}, t)$.

We define the relationship between a concrete instance and an abstract instance through the *semantic mapping* function [5], as follows:

- If $(a_1, ..., a_k, [s, e))$ is a concrete tuple, then $[\![(a_1, ..., a_k, [s, e))]\!] = \{ (a'_1, ..., a'_k, t_0) \mid s \leq t_0 < e$ where $a'_i = a_i$ if a_i is a constant, and a'_i is a point-annotated null N^{t_0} if a_i is an interval-annotated null $N^{[s,e)}\}$.
- If R is a concrete relation, then $[\![R]\!] = \bigcup_{u \in R}([\![u]\!])$.
- If I_c is a concrete instance that consists of relations $R_1, ..., R_k$, then $[\![I_c]\!] = < [\![R_1]\!], ..., [\![R_k]\!] >$.

The above definition generalizes the definition in [5] to handle null values in both views. As defined above, under the semantic mapping, each interval-annotated null $N^{[s,e)}$ corresponds to a set of point-annotated nulls $\{N^s, ..., N^{e-1}\}$ where each N^{t_0}, $t_0 \in [s, e)$, denotes a point-annotated null.

Temporal schema mapping, solution, conflicting facts. A *source-to-target temporal tuple generating dependency (s-t ttgd)*, denoted by σ_s, is a dependency of the following form:

$$\sigma_s = \forall \bar{x} \alpha(\bar{x}, t) \rightarrow \exists \bar{y} \beta(\bar{x}, \bar{y}, t),$$

where α (resp.β) is a conjunction of relational atoms (i.e. $\alpha = \alpha_1 \wedge ... \wedge \alpha_m$, $m \geq 1$ (resp. $\beta = \beta_1 \wedge ... \wedge \beta_n$, $n \geq 1$) over the source schema (respectively, the target schema), and $\bar{x}$ and $\bar{y}$ are vectors of variables and t denotes a variable that represents the temporal attribute values. We call t the *temporal variable* throughout the paper. An example of an s-t ttgd is given in Example 1.

A *temporal key constraint (tkc)*, denoted by σ_k, over a relation schema R is a constraint of the form:

$$\sigma_k = R(x_1, ..., x_m, y_1, ..., y_n, t) \wedge R(x_1, ..., x_m, y'_1, ..., y'_n, t) \rightarrow y_1 = y'_1 \wedge ... \wedge y_n = y'_n,$$

where $m \geq 0$ and $n \geq 1$ and all variables are universally quantified. The set $\{A_1, ..., A_m, T\}$ of attributes is the *temporal key* of R. A *temporal schema mapping* is a quadruple $\mathcal{M} = (R_S, R_T, \Sigma_{st}, \Sigma_k)$, where R_S and R_T are the source and target schemas, respectively; Σ_{st} is a set of s-t ttgds and Σ_k is a set of tkcs. As in SQL, (temporal) key attributes cannot be null.

We use *rhs (resp. lhs)* to refer to the right hand side (resp. left hand side) of an s-t ttgd or a tkc. We assume that there is only one temporal variable in s-t ttgds and tkcs. Multiple such variables are essential only if built-in predicates like "greater than" (i.e. $>$) are available.

When we say an abstract instance *satisfies* an s-t ttgd or a tkc (or a set of them), we refer to first order logic satisfaction and when we say a concrete instance satisfies an s-t ttgd or a tkc, it means the abstract semantics of that concrete instance satisfies the constraint.

Given an abstract source instance I_a and a temporal schema mapping $\mathcal{M}$, an *abstract solution* for I_a w.r.t $\mathcal{M}$ is an abstract target instance such that (I_a, J_a) satisfies the constraints in $\mathcal{M}$. Likewise, a *concrete solution* J_c for a concrete source instance I_c w.r.t $\mathcal{M}$ is a target instance such that (I_c, J_c) satisfies constraints in $\mathcal{M}$.

Two abstract facts are *conflicting abstract facts*, w.r.t a tkc σ_k if they agree on temporal key attribute values, but differ in other attribute values. *Conflicting concrete facts* are defined in the same way.

Formula homomorphism, abstract homomorphism, universal solution. In [7], one type of *homomorphism* is defined between two relational instances containing labeled nulls and constants, and from a conjunctive formula to a relational instance. We distinguish two types of homomorphisms: *formula homomorphism* and *abstract homomorphism* because the mapping between two point-annotated nulls in abstract homomorphisms is treated differently from the mapping of a variable in a formula to an annotated null. More formally, a *formula*

homomorphism h from the lhs of an s-t ttgd or a tkc, to an abstract instance (resp. concrete instance) I is a mapping from the variables in the lhs of the s-t ttgd or the tkc to $\mathsf{Const} \cup \mathsf{PNull}$ (resp. $\mathsf{Const} \cup \mathsf{INull}$) and from t to $\mathsf{T_P}$ (resp. $\mathsf{T_I}$) such that for every atom $R_i(\bar{x}, t)$ in α, the fact $R_i(h(\bar{x}), h(t))$ is in I. On the other hand, an *abstract homomorphism* $h : I_a \mapsto I'_a$ is a mapping from $\mathsf{Const} \cup \mathsf{PNull} \cup \mathsf{T_P}$ in an abstract instance I_a to $\mathsf{Const} \cup \mathsf{PNull} \cup \mathsf{T_P}$ in another abstract instance I'_a such that (1) $h(c) = c$, for every $c \in \mathsf{Const}$; (2) $h(t_d) = t_d$ for every time point; (3) $h(N^{t_d})$ is a constant or a point-annotated null with the context t_d; (4) for every fact $R_i(\bar{a}) \in I_a$, the fact $R_i(h(\bar{a}))$ is in I'_a.

Note that abstract homomorphisms map a point-annotated null to a point-annotated null with the same context to guarantee the assumption that point-annotated nulls have the same context as the tuple they occur in. Abstract instances I_a and I'_a are *homomorphically equivalent* if there exist abstract homomorphisms h and h' such that $h : I_a \mapsto I'_a$ and $h' : I'_a \mapsto I_a$. For example abstract instances shown in Figure 4 and Figure 5 are homomorphically equivalent.

Formula homomorphisms are used in chase steps for mapping from the lhs of an s-t ttgd or a tkc to (concrete or abstract) instances, while abstract homomorphisms are used in Theorems 7 and 8, as well as in Section 7 for proving the universality of the result of the abstract chase algorithm.

A *universal abstract solution* for a given source instance w.r.t a temporal schema mapping is a solution such that there is an abstract homomorphism from the solution to any arbitrary solution for that source instance. A *universal concrete solution* is a concrete solution whose corresponding abstract solution (obtained by the semantic mapping) is a universal abstract solution.

4 Abstract Chase

In this section, we define an abstract chase (a-chase) algorithm for the abstract view with a temporal schema mapping $\mathcal{M}$. As mentioned before, an abstract instance is a relational instance with possible infinite relations; therefore, it cannot be chased *sequentially* as in [7]. To deal with infinitely many chase steps, there are two options: define appropriate notions of fix-point or convergence, or use parallel chase. We show that our parallel chase, like the standard sequential chase, produces a universal solution (Corollary 16). The abstract chase algorithm proceeds by an s-t , denoted by a-chase$^*_{\Sigma_{st}}$, followed by a tkc round, denoted by a-chase$^*_{\Sigma_k}$. In each round, the a-chase algorithm proceeds by applying possibly infinitely many abstract chase steps in parallel. If the tkc round is successful, the result of the a-chase algorithm is an abstract solution for the given source instance w.r.t $\mathcal{M}$; otherwise, the result is a failure.

4.1 S-t ttgd Round

In the s-t ttgd a-chase round, the abstract source instance is chased with Σ_{st}. First, we define an s-t ttgd a-chase step. Then, we define an s-t ttgd a-chase round that aggregates all applicable s-t ttgd a-chase steps. In the definitions below, let I_a be a complete abstract instance.

▶ **Definition 2.** *(S-t ttgd a-chase step):* Let σ_s be an s-t ttgd in Σ_{st}. Given a formula homomorphism h from lhs of σ_s to I_a, we say that an *s-t ttgd a-chase step* can be applied to I_a with σ_s using h to get the instance K_a, denoted by $I_a \overset{\sigma_s,h}{\longmapsto} K_a$. The instance K_a is obtained by (a) extending h to h' such that each existential variable in the rhs of σ_s is

Emp

Name	Position	Company	Time
Ada	N^{2008}	IBM	2008
Ada	DBA	IBM	2008
David	N^{2008}	Intel	2008
David	Manager	Intel	2008

Figure 6 The abstract relation used in Example 4

assigned a fresh annotated null, with the temporal context $h(t)$, followed by (b) applying h' on the atoms in the rhs of σ_s.

▶ **Definition 3.** *(S-t ttgd a-chase round):* The *s-t ttgd a-chase round (a-chase$^*_{\Sigma_{st}}$) of* I_a is the union of results of all a-chase steps in Definition 2 that can be applied on the facts of I_a with all dependencies in Σ_{st}. Let J_a be the target instance obtained by the s-t ttgd round, defined as:

$$J_a = \bigcup_{\sigma_s \in \Sigma_{st}, h, I_a \xmapsto{\sigma_s, h} K_a} K_a$$

Note that J_a is an abstract solution for I_a w.r.t $\mathcal{M}' = (R_S, R_T, \Sigma_{st}, \emptyset)$ because (I_a, J_a) satisfies Σ_{st}. Corollary 14 (in Section 7) shows that J_a is a universal abstract solution for I_a w.r.t $\mathcal{M}'$.

4.2 Tkc Round

We present here the tkc a-chase round which follows the s-t ttgd a-chase round. The tkc round proceeds in a fashion similar to the s-t ttgd round: aggregating *tkc a-chase steps* on the result of the s-t ttgd round. However, the final processing is more complex. Unlike the sequential chase with equality generating dependencies (egds) in [7], where a labeled null is replaced with another labeled null or a constant (in each successful egd chase step), we show in the Example 4 that one cannot simply replace an annotated null with another value in parallel application of chase steps.

▶ **Example 4.** Consider the Emp abstract relation in Figure 6 and the following tkc:
$\forall n \forall p \forall p' \forall c \forall t\ Emp(n, p, c, t) \wedge Emp(n, p', c, t) \rightarrow p = p'$.

Notice that the sequential application of chase steps on this instance fails because DBA $\neq$ Manager. Indeed, there is no solution for this instance with the above tkc. However, the parallel application of chase steps will not fail because one of the tkc a-chase steps results in replacing N^{2008} with DBA in the first tuple and the other replaces N^{2008} with Manager in the third tuple, independently of each other. □

Instead of replacing point-annotated nulls with values during a tkc a-chase step, a set of equalities is generated that represents the replacements (Definition 5). Then, in the tkc a-chase round, we take the union of all the equalities obtained in the individual chase steps and reason about the equalities that can be deduced by considering their symmetric transitive closure (Definition 6). The abstract chase algorithm fails if an equality between two distinct constants is deduced. Otherwise, an abstract solution for the given source instance w.r.t $\mathcal{M} = (R_S, R_T, \Sigma_{st}, \Sigma_k)$ is obtained.

In the following definitions let J_a be the result of the s-t ttgd a-chase round and let $\sigma_k \in \Sigma_k$ be a tkc defined in Section 3.

▶ **Definition 5.** *(Tkc a-chase step):* Let w_1 and w_2 be two conflicting abstract facts in J_a w.r.t σ_k. If h is a formula homomorphism from lhs of σ_k to J_a such that: $w_1 = R(h(x_1), ..., h(x_m), h(y_1), ..., h(y_n), h(t))$, and
$w_2 = R(h(x_1), ..., h(x_m), h(y'_1), ..., h(y'_n), h(t))$
the tkc a-chase step can be applied on J_a with σ_k, denoted by $a\text{-}chase_{\sigma_k}(w_1, w_2)$, and the result is a set of equalities:

$$a\text{-}chase_{\sigma}(w_1, w_2) = \{h(y_i) = h(y'_i) | 1 \leq i \leq n\}.$$

▶ **Definition 6.** *(Tkc a-chase round)* : Let w_1 and w_2 be two conflicting abstract facts in J_a w.r.t σ_k. Let E_{J_a,Σ_k} be defined as follows:

$$E_{J_a,\Sigma_k} = \bigcup_{\sigma \in \Sigma_k, w_1, w_2 \in J_a} a\text{-}chase_{\sigma_k}(w_1, w_2).$$

$E_{J_a,\Sigma_k} \models x = y$, means that the symmetric transitive closure of E_{J_a,Σ_k} contains the equality $x = y$. We consider three equality cases that can be derived:

- If $E_{J_a,\Sigma_k} \models c = c'$, where c and c' are two distinct constants, then the tkc a-chase round fails and the result of the a-chase algorithm is a failure.
- If $E_{J_a,\Sigma_k} \models N^{t_0} = c$ for any point-annotated null N^{t_0}, then all occurrences of N^{t_0} in J_a will be replaced by c.
- If $E_{J_a,\Sigma_k} \models N^{t_0} = M^{t_0}$ for point-annotated nulls N^{t_0} and M^{t_0}, then one of them will be designated to replace all occurrences of the other one in J_a.

Let J'_a be the instance J_a after all replacements have been applied. Then $a\text{-}chase^*_{\Sigma_k}(J_a) = J'_a$.

Note that the order of applying the above replacements does not matter as long as all replacements induced by symmetric transitive closure of E_{J_a,Σ_k} are considered.

If the abstract chase is successful, an abstract solution is obtained for the given source instance w.r.t the given temporal schema mapping. Corollary 16 (in Section 7) shows that this solution is a universal abstract solution w.r.t $\mathcal{M}$.

5 Concrete Chase

Like the abstract chase algorithm, the concrete chase proceeds by first applying *s-t ttgd c-chase steps* in an *s-t ttgd round*, denoted by c-chase$^*_{\Sigma_{st}}$, and then *tkc c-chase steps* in a *tkc c-chase round*, denoted by c-chase$^*_{\Sigma_k}$. The definitions of c-chase steps and c-chase rounds are in the Appendix. Here, we discuss the important differences between the abstract and concrete chase algorithms.

The main difference in the definition of an s-t ttgd c-chase step from the abstract one is that the temporal attribute values are clopen intervals and interval-annotated nulls are generated for existential variables in the rhs of s-t ttgds. For example, Figure 7 shows the result of $c\text{-}chase^*_{\Sigma_{st}}$ round on the concrete source instance given in the Example 1. The other important difference is that the concrete source instance is *normalized*. Normalization is necessary to ensure that all the occurrences of the temporal variable in an s-t ttgd or a tkc map to the same clopen interval in a concrete instance. A set of concrete tuples (that might have different schemas) is *normalized* if the clopen intervals of the tuples are either equal or disjoint (i.e. no overlap in the intervals of different tuples). Note that given a normalized concrete instance I_c, each time point in $[\![I_c]\!]$ belongs to a unique interval in I_c.

Without normalization, formula homomorphisms cannot be defined from an s-t ttgd or a tkc to a concrete instance. For example, consider the concrete source instance shown in Figure 1. Assume the following s-t ttgd:

Emp

Name	Position	Company	Time
Ada	$L^{[08,10)}$	IBM	[08,10)
Ada	$E^{[10,11)}$	IBM	[10,11)
Ada	Developer	$Q^{[08,10)}$	[08,10)
Ada	DBA	$K^{[10,11)}$	[10,11)
Ada	$N^{[11,13)}$	Intel	[11,13)

Sal

Name	Position	Salary	Time
Ada	$L^{[08,10)}$	$M^{[08,10)}$	[08,10)
Ada	$E^{[10,11)}$	$U^{[10,11)}$	[10,11)
Ada	Developer	$W^{[08,10)}$	[08,10)
Ada	DBA	$P^{[10,11)}$	[10,11)
Ada	$N^{[11,13)}$	$O^{[11,13)}$	[11,13)

Figure 7 Concrete view of Emp and Sal relations obtained by the s-t ttgd c-chase round

Employee1

Name	Company	Time
Ada	IBM	[08, 10)
Ada	IBM	[10, 11)
Ada	Intel	[11, 13)

Employee2

Name	Position	Dept	Time
Ada	Developer	Computer	[08, 10)
Ada	DBA	Computer	[10, 11)

Figure 8 Normalized concrete source instance

$\sigma_s : \forall n \forall c \forall p \forall d \forall t\ Employee1(n, c, t) \wedge Employee2(n, p, d, t) \rightarrow Emp(n, p, c, t)$.
Note that the intersection of time intervals in concrete relations $Employee1$ and $Employee2$ is not empty. No formula homomorphism h can be defined from lhs of σ_s to $Employee1$ and $Employee2$ because the variable t cannot be mapped to a single time interval. However, three formula homomorphisms can be defined from lhs of σ_s to the corresponding abstract instance shown in Figure 2, such as $h' : \{n \mapsto Ada, c \mapsto IBM, p \mapsto Developer, d \mapsto Computer, t \mapsto 08\}$.

The result of normalization of the concrete instance in Figure 1 is shown in Figure 8. Now we can define two formula homomorphisms from σ_s to the normalized instance such as: $h : \{n \mapsto Ada, c \mapsto IBM, p \mapsto Developer, d \mapsto Computer, t \mapsto [08, 10)\}$.

6 Implementation of the abstract chase by the concrete chase

In this section, we show that the concrete chase is correct. This means that the result of the concrete chase is *homomorphically equivalent* to the result of the abstract chase under the semantic mapping. In the theorems of this section, let Σ_{st} and Σ_k be a set of s-t ttgds and a set of tkcs, respectively. Also, let $\sim$ denote the homomorphic equivalence between two abstract instances.

▶ **Theorem 7.** *Let I_c be a normalized complete instance. Then the result of the s-t ttgd c-chase round on I_c w.r.t Σ_{st} is homomorphically equivalent to the result of s-t ttgd a-chase round on $[\![I_c]\!]$ w.r.t Σ_{st}. That is: $[\![c\text{-}chase^*_{\Sigma_{st}}(I_c)]\!] \sim a\text{-}chase^*_{\Sigma_{st}}[\![I_c]\!]$.*

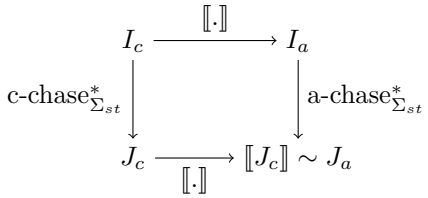

Figure 9 Correspondence between s-t ttgd c-chase and a-chase rounds

Proof Sketch: The proof can be best depicted by the commutative diagram in Figure 9. Let $J_c = c\text{-}chase^*_{\Sigma_{st}}(I_c)$ and $I_a = [\![I_c]\!]$. Let $J_a = a\text{-}chase^*_{\Sigma_{st}}(I_a)$. We need to show there is an abstract homomorphism from $[\![J_c]\!]$ to J_a (i.e. $[\![J_c]\!] \mapsto J_a$) and there is an abstract homomorphism from J_a to $[\![J_c]\!]$ (i.e. $J_a \mapsto [\![J_c]\!]$). Here we show the proof sketch for $[\![J_c]\!] \mapsto J_a$. Let $w = \beta_j(v_1, ..., v_k, t_0)$ be an abstract fact in $[\![J_c]\!]$, where $v_1, ..., v_k$ are either constants or point-annotated nulls. So there must be a concrete fact $u = \beta_j(v'_1, ..., v'_k, [s, e))$ in J_c such that $w \in [\![u]\!]$ and $t_0 \in [s, e)$. Note that since J_c is normalized, the clopen interval $[s, e)$ is the only interval which t_0 belongs to. Also, $v'_j = v_j$ $(1 \leq j \leq k)$ if v_j in w is a constant or if v_j is a point-annotated null N^{t_0}, then v'_j is an interval-annotated null $N^{[s,e)}$. The concrete fact u is generated by applying an s-t ttgd c-chase step with $\sigma_s = \forall \bar{x} \alpha(\bar{x}, t) \rightarrow \exists \bar{y} \beta(\bar{x}, \bar{y}, t)$ using homomorphism h_1 from lhs of σ_s to I_c. We build another homomorphism h_2 such that $h_2(\bar{x}) = h_1(\bar{x})$ and $h_2(t) = t_0$. Then we show that h_2 is a formula homomorphism from lhs of σ_s to $[\![I_c]\!]$ and is used in an s-t ttgd a-chase step. As a result of this a-chase step a fact $w' = \beta_j(v''_1, ..., v''_k, t_0)$ is generated. Observe that if v_j in w is a constant, $v'_j = v_j$ by the semantic mapping and $v''_j = v_j$ because of $h_2(\bar{x}) = h_1(\bar{x})$. In other words, the constants that are used in the s-t ttgd c-chase step to generate u (and are in $w \in [\![u]\!]$) are used in the s-t ttgd a-chase step with the same σ_s to generate w'. Also, all fresh point-annotated nulls that are generated to replace $\bar{y}$ are point-annotated nulls with time point t_0. Therefore, it is easy to show there is an abstract homomorphism from w to w'. After proving that for every fact in $[\![J_c]\!]$ there is an abstract homomorphism to a fact in J_a, we need to show there is an abstract homomorphism from $[\![J_c]\!]$ to J_a. We define this abstract homomorphism as the union of all the abstract homomorphisms at the facts level from $[\![J_c]\!]$ to J_a. The only remaining thing is to show that the union of abstract homomorphisms at the facts level is well-defined. Therefore, we show that each occurrence of a point-annotated null in $[\![J_c]\!]$ is mapped to the same point-annotated null in J_a.

▶ **Theorem 8.** *Let J_c be the result of s-t ttgd c-chase round. Then, $a\text{-}chase^*_{\Sigma_k}[\![J_c]\!] \sim [\![c\text{-}chase^*_{\Sigma_k}(J_c)]\!]$.*

The proof of the theorem makes use of the following lemmas. Lemma 9 shows the correspondence between conflicting concrete and abstract facts. Lemma 10 relates a tkc c-chase step to a set of tkc a-chase steps and finally Lemma 12 shows there is a correspondence between the set of equalities found by the tkc c-chase round and the tkc a-chase round. In these lemmas let J_c be the result of the s-t ttgd c-chase round which is normalized and let $J_a = [\![J_c]\!]$. Let $\sigma_k \in \Sigma_k$ be a tkc over the schema of J_c.

▶ **Lemma 9.** *Let u_1 and u_2 be two distinct concrete facts in J_c. The facts u_1 and u_2 are conflicting w.r.t σ_k if and only if all abstract facts $w_1 \in [\![u_1]\!]$ and $w_2 \in [\![u_2]\!]$ that have the same time point (i.e. $w_1[T] = w_2[T]$) are conflicting w.r.t σ_k in J_a.*

▶ **Lemma 10.** *Let u_1 and u_2 be two conflicting concrete facts in J_c w.r.t σ_k. The tkc c-chase step $c\text{-}chase_{\sigma_k}(u_1, u_2)$ can be applied to J_c with σ_k if and only if for all conflicting abstract facts $w_1 \in [\![u_1]\!]$ and $w_2 \in [\![u_2]\!]$ w.r.t σ_k, the tkc a-chase step $a\text{-}chase_{\sigma_k}(w_1, w_2)$ can be applied to J_a.*

For the next lemma, we first need to define the *semantic mapping of the set of equalities generated by tkc c-chase steps.*

▶ **Definition 11.** *(Semantic mapping of E_{J_c,Σ_k}):* Let E_{J_c,Σ_k} be the union of all the equalities found by all c-chase steps on the concrete instance J_c with a set of temporal key constraints Σ_k. We define the semantic mapping of E_{J_c,Σ_k}, denoted by $[\![E_{J_c,\Sigma_k}]\!]$, as follows:

- if $N^{[s,e)} = M^{[s,e)}$ is in E_{J_c,Σ_k}, then $\{N^s = M^s, ..., N^{e-1} = M^{e-1}\}$ is in $[\![E_{J_c,\Sigma_k}]\!]$.
- if $N^{[s,e)} = c$ is in E_{J_c,Σ_k}, where $c \in \mathsf{Const}$, then $\{N^s = c, ..., N^{e-1} = c\}$ is in $[\![E_{J_c,\Sigma_k}]\!]$.
- No other equalities are in $[\![E_{J_c,\Sigma_k}]\!]$.

▶ **Lemma 12.** *Let E_{J_c,Σ_k} be the union of all the equalities found by all tkc c-chase steps on the normalized concrete instance J_c with a set of tkcs Σ_k. Let E_{J_a,Σ_k} be union of all the equalities found by all tkc a-chase steps on $J_a = [\![J_c]\!]$ with Σ_k. Then $[\![E_{J_c,\Sigma_k}]\!] \equiv E_{J_a,\Sigma_k}$, where $\equiv$ is the logical equivalence up to renaming point-annotated nulls.*

7 Universal Solutions

In this section, we show that the result of a successful abstract chase algorithm on I_a w.r.t $\mathcal{M}$ is a universal abstract solution. As investigated in [7], universal solutions are the preferred solutions to materialize because they can be used to find certain answers to unions of conjunctive queries in polynomial time. To show the universality of an abstract solution J_a for I_a w.r.t $\mathcal{M}$, we need to show there is an abstract homomorphism from J_a to every other abstract solution J'_a for I_a w.r.t $\mathcal{M}$. In spite of universality of the solution obtained by an abstract chase on I_a w.r.t $\mathcal{M}$, it is not always possible to materialize it. Instead, because of Theorems 7 and 8, the concrete solution generated by the concrete chase algorithm on I_c w.r.t $\mathcal{M}$, such that $[\![I_c]\!] = I_a$ is materialized. This solution is called a *universal concrete solution.*

In this section, let $\mathcal{M}' = (R_S, R_T, \Sigma_{st}, \emptyset)$ and $\mathcal{M} = (R_S, R_T, \Sigma_{st}, \Sigma_k)$ be two temporal schema mappings. Recall that the abstract chase algorithm has two rounds. The universality of the solution obtained by the s-t ttgd a-chase round w.r.t $\mathcal{M}'$ is shown in Corollary 14. This corollary follows from Lemma 13 which shows there is an abstract homomorphism from the result of an s-t ttgd a-chase step on the complete abstract instance I_a to any abstract solution for I_a w.r.t $\mathcal{M}'$.

▶ **Lemma 13.** *Let $(I_a, \emptyset) \overset{\sigma_s,h}{\longmapsto} (I_a, J_i)$ be an s-t ttgd a-chase step on I_a with σ_s. Let J'_a be an instance such that (I_a, J'_a) satisfies σ_s. Then there is an abstract homomorphism from J_i to J'_a.*

▶ **Corollary 14.** *Let J_a be the result of a-chase$^*_{\Sigma_{st}}$ on I_a w.r.t $\mathcal{M}'$. Let J'_a be an instance such that (I_a, J'_a) satisfies Σ_{st}. That is J'_a is an abstract solution for I_a w.r.t $\mathcal{M}'$. Then there is an abstract homomorphism from J_a to J'_a. Therefore, J_a is a universal abstract solution w.r.t $\mathcal{M}'$.*

The universality of a tkc a-chase round is shown in Lemma 15. Corollary 16 shows the result of a-chase algorithm is a universal abstract solution.

▶ **Lemma 15.** *Let J_a be an abstract solution for I_a w.r.t $\mathcal{M}'$. Let J'_a be the result of a successful a-chase$^*_{\Sigma_k}$ round on J_a, which is an abstract solution for I_a w.r.t $\mathcal{M}$. Let J''_a be any arbitrary abstract solution for I_a w.r.t $\mathcal{M}$ and such that there exists an abstract homomorphism $h_1 : J_a \mapsto J''_a$. Then $h_1 : J'_a \mapsto J''_a$.*

▶ **Corollary 16.** *Let I_a be a source instance. Let J_a be the result of s-t ttgd a-chase round on $(I_a, \emptyset)$ with Σ_{st}. Let J'_a be the result of the tkc a-chase round on J_a with Σ_k. Then, J'_a is a universal abstract solution of I_a w.r.t $\mathcal{M}$.*

8 Query Answering

In addition to finding a universal solution for a data exchange problem, another important issue is query answering over the target schema [7]. When queries are posed over the target schema, different answers may be obtained depending on the solution that is considered. A widely used notion is the concept of *certain answers*, where the answers are the intersection of all the answers to the query on each possible solution. In this section, we first define concrete certain answers and then a method to calculate them.

▶ **Definition 17.** *Concrete certain answers*: Given a normalized concrete source instance I_c, the concrete certain answers of Q w.r.t I_c and $\mathcal{M}$, denoted by $certain_c(Q, I_c, \mathcal{M})$ are :

$$certain_c(Q, I_c, \mathcal{M}) = \bigcap\{Q(J_c) | J_c \text{ is a concrete solution for } I_c\}$$

where $Q(J_c)$ is the result of evaluating query Q on J_c treating interval-annotated nulls as distinct constants.

The definition of $certain_a(Q, I_a, \mathcal{M})$ is similar. Observe that certain answers to queries are complete tuples. Therefore, annotated nulls are used only during data exchange and do not appear in certain answers of queries.

Conjunctive queries and *unions of conjunctive queries* play an important role in relational data exchange because certain answers to this class of queries can be obtained in polynomial time [7]. Conjunctive queries are the $\exists$, $\wedge$ fragment of relational calculus. In this paper, a conjunctive query Q has the form $\exists \bar{y} \beta(\bar{x}, \bar{y}, t)$. *Unions of conjunctive queries* are a class of queries of the form $Q_1 \vee ... \vee Q_m$, where each $Q_i (1 \leq i \leq m)$ is a conjunctive query. Unions of conjunctive queries *are preserved under abstract homomorphism* [3] meaning that if there is an abstract homomorphism $h : I_a \mapsto I'_a$ and Q is a union of conjunctive queries, then if w is a complete abstract tuple in $Q(I_a)$, it is also in $Q(I'_a)$.

The following theorem claims that the certain answers of a conjunctive query Q on the concrete and abstract view are identical under the semantic mapping. The intuition behind the proof is that certain answers contain only constants and not the annotated nulls; also, the proof uses the properties of unions of conjunctive queries.

▶ **Theorem 18.** *Let $\mathcal{M}$ be a temporal schema mapping and Q be a union of conjunctive queries over the target schema. Let I_c be a concrete normalized source instance. Then we have:*

$$[\![certain_c(Q, I_c, \mathcal{M})]\!] = certain_a(Q, [\![I_c]\!], \mathcal{M}).$$

Naïve evaluation [1, 3, 9, 7] is a technique commonly used in the literature to find certain answers for unions of conjunctive queries on naïve tables. It has been shown [3, 7] that naïve evaluation of unions of conjunctive queries on a universal solution J for a relational source instance I gives certain answers. The same proof applies to universal abstract solutions, because the proof needs the definitions of certain answers and universal solutions which are the same for abstract and relational instances, modulo modifications for handling point-annotated nulls. However, for the concrete case, we need to show that naïve evaluation on a universal concrete solution will produce certain answers (Corollary 20).

Given a union of conjunctive queries Q and a concrete solution J_c for a source instance I_c and a temporal schema mapping, the naïve evaluation of Q on J_c, denoted by $Q(J_c)_{\downarrow}$, works as follows:

- Each interval-annotated null $N^{[s,e)}$ in J_c is replaced with a fresh constant $cn^{[s,e)}$ everywhere it occurs.

- Query Q is evaluated by finding all formula homomorphisms from variables in Q to J_c. In particular, the variable t is mapped to a clopen interval. The result of this step is denoted by $Q(J_c)$.
- Tuples with fresh constants are dropped from $Q(J_c)$.

Naïve evaluation of Q on an abstract solution J_a, denoted by $Q(J_a)_{\downarrow}$, works in the same way, except that each point-annotated null N^{t_0} is replaced with a fresh constant cn^{t_0}. The following theorem shows that naïve evaluation on a concrete solution produces the same answers as naïve evaluation on the corresponding abstract solution under the semantic mapping.

▶ **Theorem 19.** *Let J_c be a normalized concrete solution for a source instance I_c w.r.t $\mathcal{M}$. Let Q be a union of conjunctive queries over the target schema. Then $[\![Q(J_c)_{\downarrow}]\!] = Q([\![J_c]\!])_{\downarrow}$.*

▶ **Corollary 20.** *Let J_c be a universal concrete solution for a concrete source instance I_c w.r.t a temporal schema mapping $\mathcal{M} = (R_S, R_T, \Sigma_{st}, \Sigma_k)$. Let Q be a union of conjunctive queries over the target schema. Then $[\![certain_c(Q, I_c, \mathcal{M})]\!] = [\![Q(J_c)_{\downarrow}]\!]$.*

9 Conclusion and Future Work

In this paper, we proposed a framework for data exchange on temporal data which emphasizes on the abstract vs. concrete view of the data. We considered a basic case where source instances have a single temporal dimension, and similarly to the original data exchange framework [7], the source instances are assumed to be complete. The temporal schema mappings consist of a set of s-t ttgds and tkcs. We proposed two chase algorithms: one for concrete temporal instances and one for abstract temporal instances. The abstract chase makes it possible to define the semantics of the concrete chase and to prove that the concrete chase is correct. We defined the abstract chase as the parallel application of chase steps to avoid reasoning over infinite sequences of chase steps. We also discussed that parallel application of chase steps for temporal key constraints requires more care as not to equate one unknown value with two different constants in two chase steps. Then we defined the concrete chase and proved that its result is semantically aligned to the one achieved by the abstract chase. We showed the result of the abstract chase is a universal abstract solution and since one can implement the abstract chase by the concrete chase, the result of the concrete chase is a good candidate to be materialized and used for answering queries.

For future work, we plan to investigate our approach on real use cases. This needs an efficient implementation of normalization operation because normalization might lead to a blow-up in the number of concrete tuples if a large number of clopen intervals in the tuples within the source instance overlap each other. A normalized instance is much more fragmented than an un-normalized one, defeating the whole purpose of using intervals as a more compact representation. Furthermore, it would be nice to identify other tractable temporal constraints on the target schema. For example, egds in general or constraints with a comparison operator on time such as "If a Ph.D student graduated at time t, then there must be a time t' that they passed the candidacy examination such that $t' < t$". The challenge in the case of general egds is that the order of chase steps might matter for such dependencies because the application of one chase step might lead to the application of another one. A good starting point for considering comparison operators is the paper [18] which explores arithmetic operations in s-t tgds and tgds.

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10 Appendix

Definitions of s-t ttgd c-chase step and c-chase round

▶ **Definition 21.** *(s-t ttgd c-chase step)*: Let I_c be a complete concrete normalized instance and let σ_s be an s-t ttgd in Σ_{st}. Let h be a formula homomorphism from lhs of σ_s to I_c. The s-t ttgd σ_s *can be applied to* I_c *with homomorphism* h, denoted by $I_c \overset{\sigma,h}{\longmapsto} K_c$, and the result (i.e. J_c), is the set of facts obtained by

- extending h to h' such that each existential variable is assigned a fresh null annotated with the temporal context $h(t)$, followed by
- applying h' to the rhs of σ_s

We denote the result of applying σ_s to I_c by $I_c \overset{\sigma_s,h}{\longmapsto} K_c$.

Note that $h(t)$ is a time interval in the above definition, while it is a time point in definition 2.

▶ **Definition 22.** *(S-t ttgd c-chase round): Let I_c be a normalized concrete instance and let Σ_{st} be a set of s-t ttgds. Then the* s-t ttgd c-chase round (denoted by $c\text{-}chase^*_{\Sigma_{st}}$) of I_c is the union of results of all s-t ttgd c-chase steps that can be applied on the facts of I_c with Σ_{st}. *Let J_c be the resulting target instance after the s-t ttgd round. In short:*

$$J_c = \bigcup_{\sigma \in \Sigma_{st}, h, I_c \overset{\sigma,h}{\longmapsto} K_c} K_c$$

□

Definitions of tkc c-chase step and c-chase round

▶ **Definition 23.** *(Tkc c-chase step):* Let J_c be a normalized concrete instance. Let $\sigma_k \in \Sigma_k$ be a tkc over relation schema $R(A_1, ..., A_m, B_1, ..., B_n, T)$ in J_c where $m \geq 0$ and $n \geq 0$ (as defined in Section 3). Let u_1 and u_2 be two conflicting concrete facts w.r.t σ_k in J_c such that $u_1[T] = u_2[T] = [s, e)$ is their clopen interval. If h is a formula homomorphism from lhs of σ_k to J_c such that:

$u_1 = R(h(x_1), ..., h(x_m), h(y_1), ..., h(y_n), h(t))$ and $u_2 = R(h(x_1), ..., h(x_m), h(y'_1), ..., h(y'_n), h(t))$ then σ_k can be applied on u_1 and u_2, denoted by $c\text{-}chase_{\sigma_k}(u_1, u_2)$ and the result is a set of equalities such that:

$$c\text{-}chase_{\sigma_k}(u_1, u_2) = \{h(y_i) = h(y'_i) | 1 \leq i \leq n\}$$

Note that since J_c is normalized, t maps to a unique clopen interval, which is the interval of u_1 and u_2.

▶ **Definition 24.** *(Tkc c-chase round)*: Let J_c be a normalized concrete instance and let Σ_k be a set of tkcs. Let u_1 and u_2 be two conflicting concrete facts. Let $E_{J_c,\Sigma}$ be defined as follows:

$$E_{J_c,\Sigma_k} = \bigcup_{\sigma_k \in \Sigma_k, u_1, u_2 \in J_c} c\text{-}chase_{\sigma_k}(u_1, u_2)$$

Here we have the same cases as Definition 6 except that we deal with interval-annotated nulls. Therefore, if $E_{J_c,\Sigma} \vDash c = c'$, the result of the tkc c-chase round is a failure (case 1). Otherwise, we have a replacement of an interval-annotated null with a constant or another interval-annotated null based on the equalities implied by E_{J_c,Σ_k}. Let J'_c be the instance J_c after all replacements have been applied, then $c\text{-}chase^*_{\Sigma_k}(J_c) = J'_c$.

Note that we might have infinitely many equalities in E_{J_a,Σ_k} , but only finitely many equalities in E_{J_c,Σ_k}.

Proof of Theorem 7

Proof. Let $J_c = c\text{-}chase^*_{\Sigma_{st}}(I_c)$ and $I_a = [\![I_c]\!]$. Let $J_a = a\text{-}chase^*_{\Sigma_{st}}(I_a)$. First, we show that $[\![J_c]\!] \mapsto J_a$. Let $w = \beta_j(v_1, ..., v_k, t_0)$ be an abstract fact in $[\![J_c]\!]$, where $v_1, ..., v_k$ are either constants or point-annotated nulls. So there must be a concrete fact $u = \beta_j(v'_1, ..., v'_k, [s, e))$ in J_c such that $w \in [\![u]\!]$ and $t_0 \in [s, e)$. Based on the definition of the semantic mapping, $v'_j = v_j$ $(1 \leq j \leq k)$ if $v_j \in w$ is a constant, and if v_j is a point-annotated null N^{t_0}, then v'_j is an interval-annotated null $N^{[s,e)}$. Note that since J_c is normalized, the interval $[s, e)$ is a unique interval in J_c that the time point t_0 belongs to. The concrete fact u is in the result of the s-t ttgd c-chase round on I_c. Let

$$\sigma_s : \forall \bar{x} \alpha(\bar{x}, t) \rightarrow \exists \bar{y} \beta(\bar{x}, \bar{y}, t),$$

where $\alpha = \alpha_1 \wedge ... \wedge \alpha_m$ and $\beta = \beta_1 \wedge ... \wedge \beta_n$, be the s-t ttgd in Σ_{st} that is used in a c-chase step on I_c with a formula homomorphism h_1. Then there must be m concrete facts $u_1, ..., u_m$ in I_c such that $u_i = \alpha_i(h_1(\bar{x}), h_1(t))$, $1 \leq i \leq m$. Obviously, $h_1(t) = [s, e)$, which is the clopen interval of the facts $u_1, ..., u_m$. By extending h_1 to h'_1 as explained in Definition 21, we have $u = \beta_j(h'_1(\bar{x}), h'_1(\bar{y}), h'_1(t))$, where $h_1(\bar{x}) = h'_1(\bar{x})$, $h_1(t) = h'_1(t)$ and for each $y \in \bar{y}$, a fresh interval-annotated null with the interval $[s, e)$ is generated.
Let $q = \bigcup_{1 \leq i \leq m} [\![u_i]\!]$. Clearly $q \subseteq I_a$. We define a formula homomorphism h_2, such that $h_2(\bar{x}) = h_1(\bar{x})$, and $h_2(t) = t_0$. Now we have to show h_2 is a formula homomorphism from the lhs of σ_s to q. By applying h_2 on α_i, $1 \leq i \leq m$, we have $w_i = \alpha_i(h_2(\bar{x}), h_2(t))$. $w_i \in [\![u_i]\!]$, by construction of h_2; therefore, h_2 is a formula homomorphism from the lhs of σ_s to q. By extending h_2 to h'_2 (as explained in Definition 2), for each existential variable in $\bar{y}$ a fresh null annotated with the time point $h_2(t) = t_0$ is generated. By applying h'_2 to the rhs of σ_s, we have:

$$w' = \beta_j(h'_2(\bar{x}), h'_2(\bar{y}), h'_2(t)) = \beta_j(v''_1, ..., v''_k, t_0),$$

where v''_j $(1 \leq j \leq k)$ is either a constant or a point-annotated null with the time point t_0. Observe that, if v_j in w is a constant, $v''_j = v_j$ because $h_1(\bar{x}) = h'_1(\bar{x}) = h_2(\bar{x}) = h'_2(\bar{x})$ and if v_j in w is an annotated null such as N^{t_0}, then by construction explained above v''_j is a point-annotated null (with the time point t_0). Hence, there is an abstract homomorphism from w to w'. We have proved that there is an abstract homomorphism from each fact in $[\![J_c]\!]$ to a fact in J_a. We define an abstract homomorphism from $[\![J_c]\!]$ to J_a as the union of all the abstract homomorphisms at facts level described above. There might be different occurrences of the same point-annotated null in $[\![J_c]\!]$ because an s-t ttgd c-chase step generates the same interval-annotated null for each occurrence of an existentially quantified variable in the rhs of a σ_s and the semantic mapping produces the same set of point-annotated nulls for the same interval-annotated null. Therefore, we need to make sure that each occurrence of a point-annotated null in $[\![J_c]\!]$ maps to the same point-annotated null in J_a. In other words, the union of abstract homomorphisms at facts level is well defined. Since an s-t ttgd a-chase step, by definition, generates the same point-annotated null for each occurrence of an existentially quantified variable in the σ_s it is guaranteed all occurrences maps to the same time point. Also, note that abstract homomorphisms are identity on constants and time points, so there is no clash regarding constants and time points.

For the other direction, we need to show $J_a \mapsto [\![c\text{-}chase^*_{\Sigma_{st}}(I_c)]\!]$. Let $w = \beta_j(a_1, ..., a_k, t_0) \in a\text{-}chase^*_{\Sigma_{st}}(I_a)$, where $a_1, ..., a_k$ are either nulls annotated with t_0 or constants. The fact w is the result of a-chase on I_a. Let $\sigma (= \forall \bar{x} \alpha(\bar{x}, t) \rightarrow \exists \bar{y} \beta(\bar{x}, \bar{y}, t))$ be the s-t ttgd that was

used in an a-chase step to produce w, where $\alpha_1 \wedge ... \wedge \alpha_m = \alpha$ and $\beta_1 \wedge ... \wedge \beta_n = \beta$. This means that there is a formula homomorphism h_1 from lhs of α to m complete abstract facts $w_1, ..., w_m \in I_a$ such that $w_i = \alpha_i(h_1(\bar{x}), h_1(t))$, $1 \leq i \leq m$. Note that the facts $w_1, ..., w_m$ have the same time point t_0, that is $h_1(t) = t_0$. As the result of the a-chase step with σ_s and h_1, we have $w = \beta_j(h'_1(\bar{x}), h'_1(\bar{y}), h'_1(t))$, where h'_1 is the extension of h_1 as explained in definition 2 and β_j is an atom in β. The facts $w_1, ..., w_m$ are the result of the semantic mapping on I_c. So we have m concrete tuples in I_c such that $w_i \in [\![u_i]\!]$, $1 \leq i \leq m$. Since the instance I_c is normalized and t_0 belongs to all the clopen intervals in the tuples $u_1, ..., u_m$, we have $u_i[T] = [s, e)$, $1 \leq i \leq m$, where $[s, e)$ is a unique interval in I_c such that $t_0 \in [s, e)$. We build a formula homomorphism h_2 such that $h_2(\bar{x}) = h_1(\bar{x})$ and $h_2(t) = [s, e)$. Now we have to show, h_2 is a formula homomorphism from the lhs of α to the tuples $u_1, ..., u_m$. By applying h_2 on α, we have $\alpha_i(h_2(\bar{x}), h_2(t))$. Observe that $u_i = (h_2(\bar{x}), h_2(t))$. As the result of a c-chase step with σ_s and h_2, a fact $u = \beta_j(h'_2(\bar{x}), h'_2(\bar{y}), h'_2(t))$ will be produced where h'_2 is an extension of h_2 as described in Definition 21. The fact u has the form $\beta_j(a'_1, ..., a'_k, [s, e))$ where $a'_1, ..., a'_k$ are either constants or annotated nulls with interval $[s, e)$. Based on the definition of the semantic mapping a fact $w' = \beta_j(a''_1, ..., a''_k, t_0)$ is in $[\![u]\!]$ where $a''_j = a'_j$, $1 \leq j \leq k$ if a'_j is a constant. Or, a''_j is a point-annotated null such as N^{t_0} whenever a'_j is an interval-annotated null $N^{[s,e)}$. Observe that $a''_j = a_j$, if a_j is a constant because $h_2(\bar{x}) = h'_2(\bar{x}) = h_1(\bar{x}) = h'_1(\bar{x})$. Furthermore, if $a_j \in w$ is a point-annotated null, then a''_j is also a point-annotated null with the time point t_0 by construction of h'_2. Hence, there is an abstract homomorphism from w to w'. We have proved that there is an abstract homomorphism from each fact in J_a to a fact in $[\![J_c]\!]$. We define an abstract homomorphism from J_a to $[\![J_c]\!]$ to be the union of all abstract homomorphisms at facts level described above. There might be different occurrences of the same point-annotated null in J_a because an s-t ttgd a-chase step produces the same point-annotated null for each occurrence of an existentially quantified variable in the rhs of a σ_s. We need to make sure each occurrence of a point-annotated null in J_a maps to the same point-annotated null in $[\![J_c]\!]$ in the union of abstract homomorphisms at facts level. This is guaranteed by construction because an s-t ttgd c-chase step produces the same interval-annotated null in J_c for each occurrence of an existentially quantified variable in the rhs of the σ_s and the semantic mapping generates the same set of point-annotated nulls for each occurrence of an interval-annotated null. ◀

Proof of Lemma 9

Proof. Assume u_1 and u_2 are conflicting concrete facts w.r.t σ_k. Thus, they agree on temporal key attributes (i.e. $u_1[A_1, ..., A_m, T] = u_2[A_1, ..., A_m, T]$) but differ in some other attribute values (i.e. $\exists i, 1 \leq i \leq n$ s.t. $u_1[B_i] \neq u_2[B_i]$). Let $[s, e)$ be the time interval of u_1 and u_2 in J_c. Let w_1 and w_2 be two arbitrary facts in $[\![u_1]\!]$ and $[\![u_2]\!]$, respectively, such that $w_1[T] = w_2[T] = t_0$. Clearly, $t_0 \in [s, e)$ because of the definition of the semantic mapping. Note that since J_c is normalized, clopen interval $[s, e)$ is a unique interval t_0 belongs to. Abstract facts w_1 and w_2 also agree on temporal key attributes. Considering that the temporal key attributes cannot be null, $u_1[A_j] = u_2[A_j]$ $(1 \leq j \leq m)$ is a constant such as c. Therefore, $w_1[A_j] = w_2[A_j] = c$ based on the semantic mapping definition. Now we show w_1 and w_2 are conflicting. We distinguish all concrete conflicting cases between u_1 and u_2:

- $u_1[B_i] = c_1$ and $u_2[B_i] = c_2$, where $c_1, c_2 \in \mathsf{Const}$ and $c_1 \neq c_2$, then we have $w_1[B_i] = c_1$ and $w_2[B_i] = c_2$.
- $u_1[B_i] = N^{[s,e)}$ and $u_2[B_i] = c$ (or vice versa), then we have $w_1[B_i] = N^{t_0}$ and $w_2[B_i] = c$.
- $u_1[B_i] = N^{[s,e)}$ and $u_2[B_i] = M^{[s,e)}$, then we have $w_1[B_i] = N^{t_0}$ and $w_2[B_i] = M^{t_0}$.

All of these cases are valid based on the definition of the semantic mapping and show that w_1 and w_2 are conflicting abstract facts. For the other direction, each pair of facts $w_1 \in [\![u_1]\!]$ and $w_2 \in [\![u_2]\!]$, with the same temporal attribute $t_0 \in [s, e)$, agree on temporal key attributes $w_1[A_1, ..., A_m, T] = w_2[A_1, ..., A_m, T]$ but not on some other attribute values (i.e. $\exists i$ s.t. $w_1[B_i] \neq w_2[B_i]$). The cases where at least one of the $w_1[B_i]$ or $w_2[B_i]$ is a constant (and the other one is a point- annotated null) are straightforward. So we just discuss the case where there are different point-annotated nulls, that is $w_1[B_i] = N^{t_0}$ and $w_2[B_i] = M^{t_0}$. Then $u_1[B_i] = N^{[s,e)}$ while $u_2[B_i] = M^{[s,e)}$ which causes a conflict between the tuples u_1 and u_2. ◀

Proof of Lemma 10

Proof. First, we prove that if there is a c-chase step, such as c-chase$_\sigma(u_1, u_2)$, then there are (possibly infinitely many) a-chase steps from abstract facts in $[\![u_1]\!] \cup [\![u_2]\!]$ using the same σ_k. Let h be the formula homomorphism used in c-chase$_\sigma(u_1, u_2)$ from lhs of σ_k to J_c such that: $u_1 = R(h(x_1), ..., h(x_m), h(y_1), ..., h(y_n), h(t))$ and $u_2 = R(h(x_1), ..., h(x_m), h(y'_1), ..., h(y'_n), h(t))$

Based on Lemma 9 all abstract facts $w_1 \in [\![u_1]\!]$ and $w_2 \in [\![u_2]\!]$ that agree on the temporal attribute are conflicting abstract facts. Let t_0 be an arbitrary time point in $[s, e)$, we build formula homomorphism h_{t_0} as follows:

$$h_{t_0}(z) = \begin{cases} h(z) & \text{if } h(z) \text{ is a constant and } z \text{ is a non-temporal variable} \\ N^{t_0} & \text{if } h(z) \text{ is an interval-annotated null } N^{[s,e)} \\ t_0 & \text{if } h(z) = [s, e) \end{cases}$$

Now we need to show h_{t_0} is a formula homomorphism from σ_k to $w_1 \in [\![u_1]\!]$ and $w_2 \in [\![u_2]\!]$, such that $w_1[T] = w_2[T] = t_0$. Wlog, we just consider w_1. Applying h to σ_k, the following fact is obtained

$$R(h(x_1), ..., h(x_m), h(y_1), ..., h(y_n), h(t)) = R(c_1, ..., c_m, v_1, ..., v_n, [s, e))$$

where $c_1, ..., c_m$ are constants, v_j $(1 \leq j \leq n)$ is either constant or an interval-annotated null. If we apply h_{t_0} to σ_k, the result, based on definition of h_{t_0}, is

$$R(h_{t_0}(x_1), ..., h_{t_0}(x_m), h_{t_0}(y_1), ..., h_{t_0}(y_n), h_{t_0}(t)) = R(c_1, ..., c_m, v'_1, ..., v'_n, t_0)$$

where v'_j, $1 \leq j \leq n$ is either a constant or a point-annotated null. Observe that $v'_j = v_j$, if v_j is constant and $v'_j = N^{t_0}$ if $v_j = N^{[s,e)}$. Therefore, $w_1 = R(c_1, ..., c_m, v'_1, ..., v'_n, t_0)$ and σ_k can be applied on w_1 and w_2 (i.e. a-chase$_{\sigma_k}(w_1, w_2)$).

In the other direction (i.e. only if), for each pair of conflicting abstract facts $w_1 \in [\![u_1]\!]$ and $w_2 \in [\![u_2]\!]$, with the same time point $t_0 \in [s, e)$, there is an a-chase step (i.e. a-chase$_{\sigma_k}(w_1, w_2)$). Let h_{t_0} be the homomorphism used in a-chase$_\sigma(w_1, w_2)$ step. Note that each time point can belong to only one interval in a normalized instance. Based on Lemma 9, we have two conflicting concrete facts u_1 and u_2 with the interval $[s, e)$. We define a formula homomorphism h as follows

$$h(z) = \begin{cases} h_{t_0}(z) & \text{if } h_{t_0}(z) \text{ is a constant and } z \text{ is a non-temporal variable} \\ N^{[s,e)} & \text{if } h_{t_0}(z) \text{ is an annotated null } N^{t_0} \\ [s, e) & h_{t_0}(z) = t_0 \end{cases}$$

Now we need to show h is a formula homomorphism from σ_k to u_1 and u_2. Wlog, we show this for u_1. When we apply h_{t_0} to σ_k, we get the tuple

$$w_1 = R(h_{t_0}(x_1), ..., h_{t_0}(x_m), h_{t_0}(y_1), ..., h_{t_0}(y_n), h_{t_0}(t)) = R(c_1, ..., c_m, v'_1, ..., v'_n, t_0)$$

where $c_1, ..., c_m$ are constants and $v'_j (1 \leq j \leq n)$ is either a constant or an annotated null. Now by applying h on σ_k the following fact is obtained

$$R(h(x_1), ..., h(x_m), h(y_1), ..., h(y_n), h(t)) = R(c_1, ..., c_m, v_1, ..., v_n, [s, e))$$

where v_j, $1 \leq j \leq n$, is either a constant or an interval-annotated null. Observe that $v_j = v'_j$ if v'_j is a constant and $v_j = N^{[s,e)}$ if $v'_j = N^{t_0}$. Therefore, $u_1 = R(c_1, ..., c_m, v_1, ..., v_n, [s, e))$. Therefore, we can apply σ_k on u_1 and u_2 (i.e. c-chase$_{\sigma_k}(u_1, u_2)$).

◀

Proof of Lemma 12

Proof. Each equality in E_{J_c,Σ_k}, such as $N^{[s,e)} = val$, is due to a tkc c-chase step on conflicting concrete facts u_1 and u_2, such that $u_1[T] = u_2[T] = [s, e)$. Based on Lemma 9 and Lemma 10, we know there is a tkc a-chase step for each pair of conflicting abstract facts w_1 and w_2 w.r.t σ_k such that $w_1[T] = w_2[T] = t_0$ and $t_0 \in [s, e)$. Therefore, based on Definition 11 for each equality in E_{J_c,Σ_k}, there is an equivalent equality in E_{J_a,Σ_k} up to renaming the point-annotated nulls.
The other direction is straightforward as well. Therefore, $[\![E_{J_c,\Sigma_k}]\!] \equiv E_{J_a,\Sigma_k}$. ◀

Proof of Theorem 8

Proof. This proof is very similar to the proof of Theorem 7. First we show $[\![c\text{-}chase^*_{\Sigma_k}(J_c)]\!] \mapsto a\text{-}chase^*_{\Sigma_k}[\![J_c]\!]$. Let w be a fact in $[\![c\text{-}chase^*_{\Sigma}(J_c)]\!]$ such that $w = R(a_1, ..., a_m, t_0)$, where a_i is either a constant or a point-annotated null. This means that there is a concrete fact u in $J'_c = c\text{-}chase^*_{\Sigma_k}(J_c)$ such that $w \in [\![u]\!]$. Let $u = R(b_1, ..., b_m, [s, e))$, where b_i is either a constant or an interval-annotated null. By definition of the semantic mapping, we have

- if a_i is a constant, b_i is a constant as well.
- if a_i is a point-annotated null such as N^{t_0}, b_i is an interval-annotated null such as $N^{[s,e)}$.
- $t_0 \in [s, e)$ for some positive integers s and e.

The concrete fact u is the result of $c\text{-}chase^*_{\Sigma_k}$ (Definitions 23 and 24) on concrete instance J_c where Σ_k is a set of temporal key constraints. We distinguish two cases:

- The concrete fact u is in J_c and it is not conflicting with any other concrete tuple with respect to Σ_k. Therefore, it will remain intact in J'_c after applying parallel chase $c\text{-}chase^*_{\Sigma_k}$.
- Some interval-annotated nulls in the fact u were replaced by constants or other interval-annotated nulls based on the equalities in E_{J_c,Σ_k} during a successful $c\text{-}chase^*_{\Sigma_k}$ step.

Now we need to show in either case, there is a fact w' in $a\text{-}chase^*_{\Sigma_k}[\![J_c]\!]$ that is equivalent to w up to renaming point-annotated nulls (without changing their context)

Case(a): Since u is a concrete fact in J_c and $w \in [\![u]\!]$ and we are a-chasing $[\![J_c]\!]$ with the same set Σ_k of tkcs, the abstract fact w is not conflicting with any other fact in $[\![J_c]\!]$ w.r.t any $\sigma_k \in \Sigma_k$. Hence, w' is equal to $w \in J_a = a\text{-}chase^*_{\Sigma_k}([\![J_c]\!])$.

Case(b): There is a fact u' in J_c such that u can be obtained from u' by replacing its interval-annotated nulls by either constants or other interval-annotated nulls. These replacements are inferred during parallel c-chase by E_{J_c,Σ_k}. Based on Lemma 12 we have $[\![E_{J_c,\Sigma_k}]\!] \equiv E_{J_a,\Sigma_k}$, thus, the point-annotated nulls that are replaced in $w' \in a\text{-}chase^*_{\Sigma_k}[\![u']\!]$ are based on an equivalent set of equalities (i.e. E_{J_a,Σ_k}). Note that fact w' is not identical to w because we do not have control over the arbitrary point-annotated null that is chosen to replace the others which are equivalent to it, but they have the same context. Therefore, there is an abstract homomorphism from w' to w. Proof of $a\text{-}chase^*_{\Sigma_k}(J_a) \mapsto [\![c\text{-}chase^*_{\Sigma_k}(J_c)]\!]$ is similar. ◀

Proof of Lemma 13

Proof. The proof is same as proof of *case 1 in Lemma 3.4* in [7], which shows there is a homomorphism from the result of an arbitrary chase step in the sequence of chase steps to any instance that satisfies σ_s. Lemma 13 is a special case of Lemma 3.4, where the initial instance is $(I_a, \emptyset)$. Therefore there is no need for the assumption that there exists a homomorphism from $(I_a, \emptyset)$ to the instance (J'_a in our case) that satisfies σ_s. In other words, an abstract homomorphism from $(I_a, \emptyset)$ to (I_a, J'_a) is the identity mapping (same as homomorphisms in [7]). ◀

Proof of Lemma 15

Proof. Wlog, we assume all tkcs in Σ_k have one equality on rhs, that is:

$$\sigma_k : R(x_1, ..., x_m, y_1, t) \wedge R(x_1, ..., x_m, y_2, t) \rightarrow y_1 = y_2$$

Let $R(c_1, ..., c_m, v, t_0)$ be a fact in J'_a, where $c_1, ..., c_m$ are constants and v is either a constant or a point-annotated null. We need to show that $R(h_1(c_1), ..., h_1(c_m), h_1(v), h_1(t_0))$ is in J''_a. If $R(v_1, ..., v_n, t_0) \in J_a$, then since $h_1 : J_a \mapsto J''_a$, we have $R(h_1(v_1), ..., h_1(v_n), h_1(t_0)) \in J''_a$. In this case, the above fact has not been modified during *a-chase*$^*_{\Sigma_k}$ round. Otherwise, by the definition of a tkc a-chase round, there is a fact $R(c_1, ..., c_m, v', t_0)$ in J_a and the equality $v' = v$ is derived form E_{J_a,Σ_k}. This equality is either generated in one tkc a-chase step or is derived by symmetric transitive closure of E_{J_a,Σ_k}. We will discuss each case below:

- Equality $v' = v$ is generated in one tkc a-chase step. Then there is a formula homomorphism h from the lhs of σ_k to J_a such that $h(y_1) = v'$ and $h(y_2) = v$. Since $h_1 : J_a \mapsto J''_a$, the abstract homomorphism h_1 is defined for v and v', that is $h_1(v)$ and $h_1(v')$ are in J''_a. Also, since J''_a satisfies σ_k it follows that $h_1(v)$ is equal to $h_1(v')$ in J''_a. Therefore, $R(h_1(c_1), ..., h_1(c_m), h_1(v'), h_1(t_0)) \in J''_a$.
- Equality $v' = v$ is derived by symmetric transitive closure of E_{J_a,Σ_k}. In this case, at least two different formula homomorphisms are used that map variables in the lhs of at least two tkcs in Σ_k to J_a. In particular v and v' are in J_a. Since $h_1 : J_a \mapsto J''_a$, the abstract homomorphism h_1 is defined for $h_1(v_i)$ and $h_1(v'_i)$. Moreover, since J''_a is a solution for I_a w.r.t $\mathcal{M}$, we have $h_1(v_i)$ is equal to $h_1(v'_i)$ in J''_a and $R(h_1(c_1), ..., h_1(c_m), h_1(v'), h_1(t_0)) \in J''_a$.

◀

Proof of Theorem 18

Proof. First, we show $[\![certain_c(Q, I_c, \mathcal{M})]\!] \subseteq certain_a(Q, [\![I_c]\!], \mathcal{M})$. Let w be a tuple in $[\![certain_c(Q, I_c, \mathcal{M})]\!]$, so w is a complete abstract fact. There must be a complete concrete fact u in $certain_c(Q, I_c, \mathcal{M})$ such that $w \in [\![u]\!]$. Based on Definition 17, $u \in \bigcap\{Q(J_c) | J_c \text{ is a concrete solution for } I_c\}$. Let J'_c be the concrete solution produced by concrete chase algorithm on I_c w.r.t $\mathcal{M}$. Therefore, $u \in Q(J'_c)$ and $w \in [\![Q(J'_c)]\!]$. Let J_a be the abstract solution obtained by the abstract chase algorithm on $[\![I_c]\!]$ w.r.t $\mathcal{M}$. Based on Theorems 7 and 8 we know that J_a is homomorphically equivalent to $[\![J'_c]\!]$. Since w is a complete fact in $[\![Q(J'_c)]\!]$, unions of conjunctive queries are preserved under abstract homomorphisms, and abstract homomorphisms are identities on constants, w is in $Q(J_a)$. The solution J_a is a universal abstract solution (Corollary 16). Therefore, w is in the result of evaluating Q on every abstract solution for $[\![I_c]\!]$, which means $w \in certain_a(Q, [\![I_c]\!], \mathcal{M})$.

For the other direction, we have to prove $certain_a(Q, [\![I_c]\!], \mathcal{M}) \subseteq [\![certain_c(Q, I_c, \mathcal{M})]\!]$. Let $w \in certain_a(Q, [\![I_c]\!], \mathcal{M})$, which means w is a complete abstract tuple in:

$$\bigcap\{Q(J_a) | J_a \text{ is an abstract solution for } [\![I_c]\!]\}.$$

Let J'_a be the abstract solution obtained by abstract chase algorithm on $[\![I_c]\!]$ w.r.t $\mathcal{M}$. We have $w \in Q(J'_a)$. Note that J'_a is a universal abstract solution. Let J_c be the concrete solution produced by concrete chase on I_c w.r.t $\mathcal{M}$. Based on Theorems 7 and 8, we know that $w \in [\![Q(J_c]\!]$. Now we have to show that w is in $[\![Q(J'_c)]\!]$, where J'_c is any arbitrary concrete solution for I_c. Suppose $w \notin [\![Q(J'_c)]\!]$. It is obvious that $[\![J'_c]\!]$ is also a solution for $[\![I_c]\!]$ w.r.t $\mathcal{M}$. Since J_a is a universal abstract solution, there must be an abstract homomorphism from J_a to $[\![J'_c]\!]$. However, it is a contradiction with universality of J_a if $w \in Q(J_a)$ and not in $[\![J'_c]\!]$. Therefore w must be in $[\![J'_c]\!]$. ◀

Naïve evaluation for abstract instances

Naïve evaluation [1, 3, 9, 7] is a technique commonly used in the literature to find certain answers for unions of conjunctive queries on naïve tables (i.e. tables with labeled nulls). Let J_a be a universal abstract solution for a source instance I_a and a schema mapping. Let Q be a union of conjunctive queries of the form $Q_1 \vee ... \vee Q_m$, where each Q_i $(1 \leq i \leq m)$ is a conjunctive query (i.e. $\exists \bar{y} \beta(\bar{x}, \bar{y}, t)$), where β is a conjunction of atomic formulas(i.e. $\beta = \beta_1 \wedge ... \wedge \beta_n$) over the target schema. In order to calculate the result of Q on the abstract instance J_a with naïve evaluation, denoted by $Q(J_a)_{\downarrow}$, the following steps should be taken:

- All point-annotated nulls are treated as distinct fresh constants. In particular, the point-annotated null N^{t_0} is replaced with a fresh constant cn^{t_0} everywhere it occurs in J_a.
- Query Q is evaluated by finding all formula homomorphisms from variables in Q to J_a. In particular, the variable t is mapped to a time point. The result of this step is denoted by $Q(J_a)$.
- Tuples with fresh constants are dropped from $Q(J_a)$.

Proof of Theorem 19

Proof. Wlog, we prove the theorem for one disjunct of Q'. Let Q be an arbitrary disjunct in Q'. This equality is proved by first showing that $[\![Q(J_c)_{\downarrow}]\!] \subseteq Q([\![J_c]\!])_{\downarrow}$ and then $Q([\![J_c]\!])_{\downarrow} \subseteq [\![Q(J_c)_{\downarrow}]\!]$. Like the previous proofs, for each direction, we take a tuple in the left hand side and show it must be in the right hand side as well and vice versa.

For the first part, let $w = (a_1, ..., a_k, t_0)$ be a complete abstract tuple in $[\![Q(J_c)_{\downarrow}]\!]$. This means there should be a complete concrete tuple $u = (a'_1, ..., a'_k, [s, e))$ in $Q(J_c)_{\downarrow}$ such that $w \in [\![u]\!]$. So, $t_0 \in [s, e)$. Furthermore, since naïve evaluation discards tuples with fresh constants (that were nulls), all non-temporal values in w and u must be constants. That is, $a_i = a'_i$ and they are constants, $1 \leq i \leq k$.

Having $u \in Q(J_c)_{\downarrow}$ means that there is a formula homomorphism h from the variables in Q to the target instance J_c such that $u = (h(\bar{x}), h(t))$. We need to show that there is a formula homomorphism h' from Q to $[\![J_c]\!]$ such that $w = (h'(\bar{x}), h'(t))$. We build h' as follows:

$$h'(z) = \begin{cases} h(z) & \text{if } h(z) \text{ is a constant and } z \text{ is a non-temporal variable} \\ nc^{t_0} & \text{if } h(z) \text{ is a fresh constant } nc^{[s,e]} \\ t_0 & \text{if } h(z) = [s, e) \end{cases}$$

Observe that $h'(\bar{x}) = h(\bar{x})$ because $h(\bar{x})$ consists of constants. Now we have to show h' is a formula homomorphism from Q to $[\![J_c]\!]$. Let $\beta_i(x_1, ..., x_k, y_1, ..., y_m, t)$ be an arbitrary atom in

Q that maps to a concrete fact $u' = \beta_i(a_1, ..., a_k, v_1, ..., v_m, [s, e))$ in J_c with homomorphism h, where $a_i, 1 \leq i \leq k$ are constants and v_j $(1 \leq j \leq m)$ is either a constant or a fresh constant. That is, $u' = (h(x_1), ..., h(x_k), h(y_1), ..., h(y_m), h(t))$. Using the homomorphism h' on β_i gives a fact $w' = \beta_i(h'(x_1), ..., h'(x_k), h'(y_1), ..., h'(y_m), h'(t))$ which according to definition of h' is $\beta_i(a_1, ..., a_k, v'_1, ..., v'_m, t_0)$, where $v'_j = v_j$ if v_j is a constant and $v'_j = nc_j^{t_0}$ if v_j is a fresh constant $nc_j^{[s,e)}$. Observe that $w' \in [\![u']\!]$; therefore, the homomorphism h' is a mapping from Q to $[\![J_c]\!]$. Moreover, $w = (h'(\bar{x}), h'(t))$, hence it is in $Q([\![J_c]\!])_{\downarrow}$.

For the other direction, we have to show $Q([\![J_c]\!])_{\downarrow} \subseteq [\![Q(J_c)_{\downarrow}]\!]$. Let $w = (a_1, ..., a_k, t_0)$ be a complete abstract tuple in $Q([\![J_c]\!])_{\downarrow}$. Therefore, there is a formula homomorphism h' from variables in Q to $[\![J_c]\!]$ such that $w = (h'(\bar{x}), h'(t))$. Since J_c is normalized, let $[s, e)$ be the unique clopen interval in J_c that contains t_0. We need to show that there is a formula homomorphism h from variables in Q to J_c such that $u = (h(\bar{x}, t))$ is in $Q(J_c)_{\downarrow}$ and $w \in [\![u]\!]$. In other words, $u = (a_1, ..., a_k, [s, e))$ such that $t_0 \in [s, e)$. We build a homomorphism h from variables in Q to J_c as follows:

$$h(z) = \begin{cases} h'(z) & \text{if } h'(z) \text{ is a constant and } z \text{ is a non-temporal variable} \\ nc^{[s,e)} & \text{if } h'(z) \text{ is a fresh constant } nc^{t_0} \\ [s, e) & \text{if } h'(z) = t_0 \end{cases}$$

Observe that $h(\bar{x}) = h'(\bar{x})$, because $h'(\bar{x})$ is mapped to constants. Let $\beta_i(x_1, ..., x_k, y_1, ..., y_m, t)$ be an arbitrary atom in Q. Under h', the atom maps to an abstract fact w' in $[\![J_c]\!]$ such that $w' = \beta_i(a_1, ..., a_k, v_1, ..., v_m, t_0)$, where $a_i, 1 \leq i \leq k$ are constants and v_j, $1 \leq j \leq m$ are constants or fresh constants. Under h, the atom β_i is $u' = \beta_i(a_1, ..., a_k, v'_1, ..., v'_m, [s, e))$, where $v'_j = v_j$ if v_j is a constant and $v'_j = nc_j^{[s,e)}$ if v_j is a fresh constant $nc_j^{t_0}$.

Observe that $w' \in [\![u']\!]$. Since J_c is normalized, there is a unique clopen interval among all time values in J_c that contains t_0. As mentioned earlier, this interval is $[s, e)$. Hence, if w' is a fact in $[\![J_c]\!]$, then u' is the unique fact in J_c such that $w \in [\![u']\!]$. Since there is only one temporal variable t and $\beta_i(h'(\bar{x}), h'(\bar{y}), h'(t))$ is in $[\![J_c]\!]$, it follows that $\beta_i(h(\bar{x}), h(\bar{y}), h(t))$ is a fact in J_c. This shows h is a formula homomorphism from Q to J_c.

Hence, $u = (h(\bar{x}), h(t))$ is in $Q(J_c)_{\downarrow}$, and so we have $w \in [\![Q(J_c)_{\downarrow}]\!]$. Since the disjuncts of Q' are independent and Q is an arbitrary disjunct in Q', it is concluded that $[\![Q'(J_c)_{\downarrow}]\!] = Q'([\![J_c]\!])_{\downarrow}$. ◀

Proof of Theorem 20

Proof. Naïve evaluation of unions of conjunctive queries on a universal abstract solution $[\![J_c]\!]$ gives certain answers. The proof is the same as in [3, 7]. Therefore,

$$certain_a(Q, [\![I_c]\!], \mathcal{M}) = Q([\![J_c]\!])_{\downarrow}$$

From Theorem 18:

$$[\![certain_c(Q, I_c, \mathcal{M})]\!] = Q([\![J_c]\!])_{\downarrow}$$

By replacing $Q([\![J_c]\!])_{\downarrow}$ with $[\![Q(J_c)_{\downarrow}]\!]$ from Theorem 19:

$$[\![certain_c(Q, I_c, \mathcal{M})]\!] = [\![Q(J_c)_{\downarrow}]\!]$$

◀