Logical Time

CSE 486: Distributed Systems

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Time Synchronization

As we have seen, time synchronization is hard.

Often, what we actually care about is causality, not time.

Could some event have caused another event?

If we can establish this, we may not need physical time!

Logical Clocks

Logical clocks were first introduced by Lamport in 1978 [2].

They address ordering without requiring time synchronization.

Not all problems can be solved with logical clocks!

Required Readings

This lecture has another required reading [1].

You are expected to keep up with required readings.

You should have already read all previous required readings!

They may show up on the Midterm/Final, such as:

A centralized failure detector model reduces communication overhead, but violates the end-to-end-principle. Explain why it does not preserve the end-to-end principle, and discuss the trade-offs that it makes in terms of communication complexity and robustness versus end-to-end failure detection.

This is an upper level course, read and think! Ask questions!

Event Ordering

Logical clocks directly encode the happens before relationship.

This establishes three possible conditions for events e_1 and e_2:

- e_1 happens before e_2
- e_2 happens before e_1
- Neither event happens before the other, they are concurrent

This is a partial ordering.

Notation

If e_1 happens before e_2, we say $e_1 \rightarrow e_2$.

If e_1 does not happen before e_2, we say $e_1 \not\rightarrow e_2$.

Note that this does not mean that e_2 happens before e_1!

If $e_1 \not\rightarrow e_2$ and $e_2 \not\rightarrow e_1$, then e_1 and e_2 are concurrent.

Events in a Process

The events within a single process form a total ordering.

Every event in the process happens before the next, sequentially.

For every event within a process, either $p_1 \rightarrow p_2$ or $p_2 \rightarrow p_1$.

This implies that processes have a single thread of control.

We conventionally number these events in numeric order. (That is, $p_1 \rightarrow p_2$.)

Messages

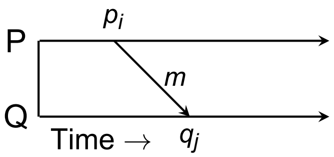
Sending and receipt of messages are events.

Sending a message happens before the message is recieved.

Suppose that:

- Message m is sent from process P as event p_i
- Process Q receives m as event q_j

Therefore $p_i \rightarrow q_j$.

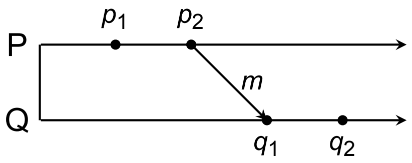


Transitivity

Happens before is transitive.

If $e_i \rightarrow e_j$ and $e_j \rightarrow e_k$, then $e_i \rightarrow e_k$.

This allows messages to order events between processes.



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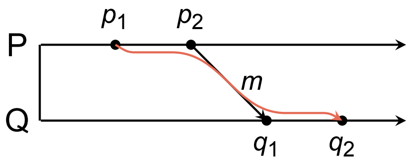
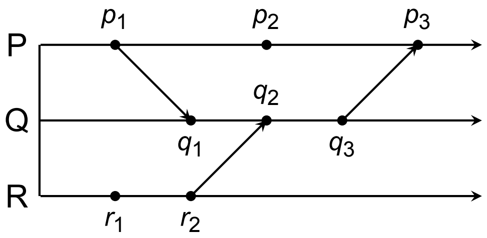


$p_1 \rightarrow q_2$

Concurrent Events

Concurrent events can only occur between processes.



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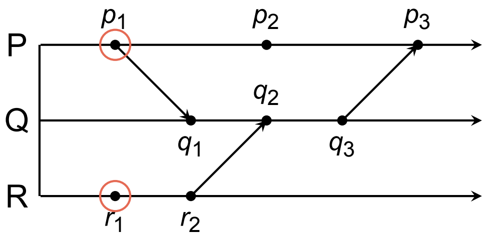

r_1 and p_1 are concurrent.

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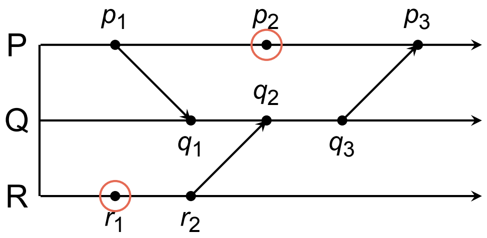

r_1 and p_2 are concurrent.

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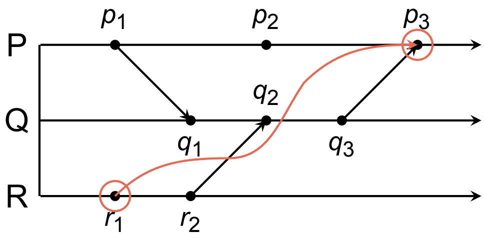


$r_1 \rightarrow p_3$.

Lamport Clocks

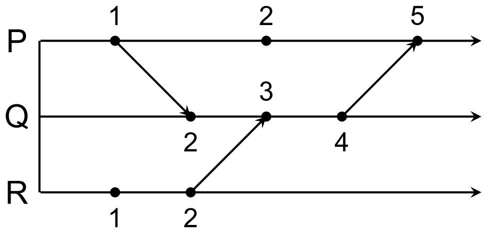
Lamport clocks number events with a logical timestamp.

The rules are simple:

- Every process starts with a timestamp of 1.
- Every time a process takes an action, it increments its timestamp.
- Sending a message is an action.
- Messages include the timestamp of their action.
- Receiving a message is an action.
- After reception, processes set their timestamp to the maximum of their local timestamp and the message timestamp plus 1.

Timestamp Example

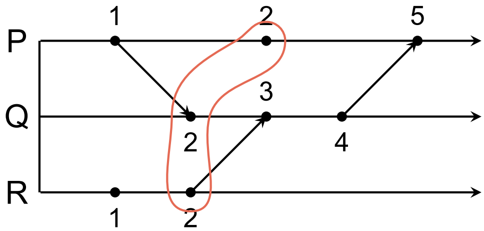
These timestamps follow the Lamport clock rules.



If $e_1 \rightarrow e_2$, the timestamp of e_1 is numerically less than the timestamp of e_2.

Timestamp Example

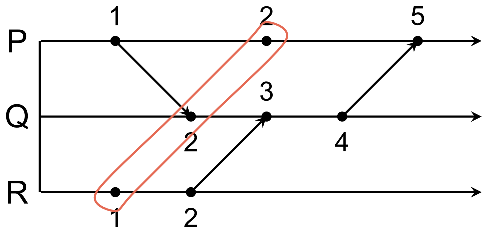
Note that concurrency is ambiguous in the timestamps.



These points are concurrent.

Timestamp Example

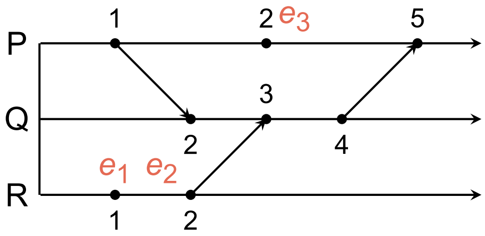
Note that concurrency is ambiguous in the timestamps.



So are these!

Timestamp Example

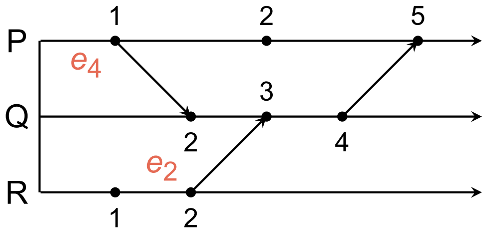
Note that concurrency is ambiguous in the timestamps.



e_1 and e_2 are both concurrent with e_3, but $e_1 \rightarrow e_2$!

Timestamp Example

Note that concurrency is ambiguous in the timestamps.



…and e_4 and e_2 are concurrent, too!

Causality

Lamport clocks approximate causality:

If the timestamp of e_1 < the timestamp of e_2, then e_1 could have caused e_2.

If $e_1 > e_2$, then e_1 could not have caused e_2.

The mapping is not perfect, with false positives.

There are no false negatives.

Vector Clocks

Vector clocks associate more than one timestamp with an event [3].

Each process has its own timestamp.

Each event is timestamped with the causality of every process.

This provides a tighter mapping with fewer false positives.

There are still no false negatives.

Vector Clock Rules

Every process P_i keeps a vector of clock values.

There is one vector entry for each process.

P_i can increment only the i^{th} entry.

Each process takes the max of every vector position on message receipt.

Vector Clock Ordering

For vector $v = \langle p_0, \ldots, p_n \rangle$ and another u:

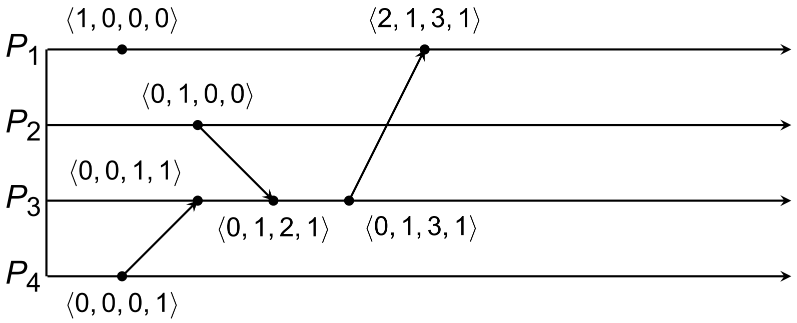
$$u = v \quad \text{iff} \quad \forall_{i=0}^{n} \quad u[i] = v[i]$$

$$u \leq v \quad \text{iff} \quad \forall_{i=0}^{n} \quad u[i] \leq v[i]$$

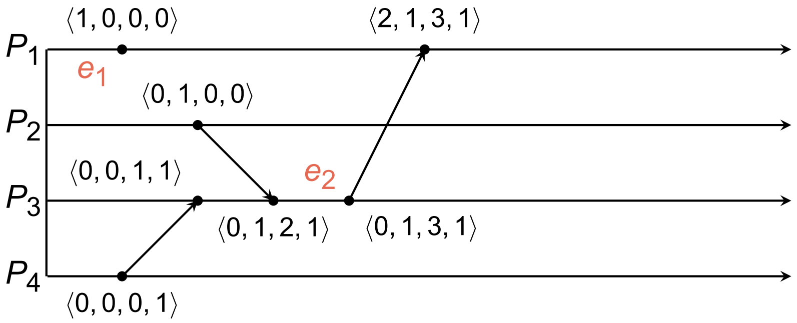
$$u < v \quad \text{iff} \quad u \leq v \text{ and } u \neq v$$

$$u \parallel v \quad \text{iff} \quad \neg(u < v) \text{ and } \neg(v < u)$$

Vector Clock Example


$\langle 1, 0, 0, 0 \rangle$
$\langle 2, 1, 3, 1 \rangle$
P_1
$\langle 0, 1, 0, 0 \rangle$
P_2
$\langle 0, 0, 1, 1 \rangle$
P_3
$\langle 0, 1, 2, 1 \rangle$
$\langle 0, 1, 3, 1 \rangle$
P_4
$\langle 0, 0, 0, 1 \rangle$

Vector Clock Example


$\langle 1, 0, 0, 0 \rangle$
$\langle 2, 1, 3, 1 \rangle$
P_1
e_1
$\langle 0, 1, 0, 0 \rangle$
P_2
$\langle 0, 0, 1, 1 \rangle$
e_2
P_3
$\langle 0, 1, 2, 1 \rangle$
$\langle 0, 1, 3, 1 \rangle$
P_4
$\langle 0, 0, 0, 1 \rangle$

e_1 is unambiguously concurrent with e_2
because $\langle 1, 0, 0, 0 \rangle \parallel \langle 0, 1, 3, 1 \rangle$

Disadvantages of Vector Clocks

Vector clocks have better precision than Lamport clocks.

They identify concurrent events more precisely.

However, they require more state.

For large numbers of processes they may be impractical.

Total Ordering

Both Lamport and vector clocks can provide a total ordering.

This requires breaking ties between concurrent events.

Some arbitrary mechanism can be used; *e.g.*:

- process IDs for Lamport clocks
- numerical order for vector clocks
  (For example: $\langle 1, 2, 3, 4 \rangle$ comes before $\langle 1, 3, 2, 5 \rangle$)
- Supplementary physical timestamps

This total ordering is not physical time ordering!

Summary

- Logical clocks track causality of events
- Lamport clocks use a single integer to define causality
- Vector clocks provide greater precision than Lamport clocks, but require more state
- Logical clock orderings can be partial or total

Next Time ...

- ...

References I

Required Readings

[1] Ajay D. Kshemkalyani and Mukesh Singhal. *Distributed Computing: Principles, Algorithms, and Systems*. Chapter 2: 2.1–2.3, 2.6; Chapter 3: 3.1–3.4. Cambridge University Press, 2008. ISBN: 978-0-521-18984-2.

Optional Readings

[2] Leslie Lamport. “Time, Clocks, and the Ordering of Events in a Distributed System”. In: 21.7 (July 1978). Ed. by R. Stockton Gaines, pp. 558–565. URL: https://dl-acm-org.gate.lib.buffalo.edu/doi/pdf/10.1145/359545.359563.

References II

[3] Friedemann Mattern. “Virtual Time and Global States of Distributed Systems”. In: *Proceedings of the Workshop on Parallel and Distributed Algorithms*. Elsevier Science Publishers B.V., Oct. 1988, pp. 215–226. URL: `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.1068.1331`.