# Homework #2

Due: 3/12/23 @ 11:59pm

**Content Covered:** Logical Equivalence, Predicates, Quantifiers, Logical Reasoning

## Submission Instructions

Submit your completed homework to UBLearns electronically in PDF format. Any submissions that are not a PDF or not a legible PDF will not receive credit. We need to be able to read your submission to be able to grade your work. Your write-up should contain enough information from the problem so that a reader doesn't need to return to the text to know what the problem is (it is a good habit to rewrite each problem prior to solving it). There is no general rule for how much information from the problem to include, but it should be possible to read your homework and ascertain what the problem was and what your solution is accomplishing.

When writing up the solution, you may hand write the solutions and submit a scanned PDF, or write up the solutions electronically and convert them to a PDF. If you hand write your solutions, make sure that you write clearly and your writing is legible. Double check your scans to make sure that your scanned copy is legible. After you submit your work, make sure the file is visible. Download your submitted copy, open it, and see whether you submitted the correct file and your submitted file has not been corrupted during the upload.

You are able to upload your submission multiple times. Only the last file will be graded. Keep in mind that if your completed work consists of multiple pages and you submit a separate file for each page, only the last file submitted will be graded. In this case, only one page of your submission would be graded. **You are responsible for making sure that your submission goes through as intended.**

Your submitted work must be your own. Please review the course Academic Integrity Policy as outlined in the syllabus. **Failure to adhere to this policy will result in an F in the course.**

## Late Policy

Late homework will be accepted up to 1 day late for a penalty of 25% of the total points. For example, if the homework is worth 100 points and you submit it one day late, you will receive the maximum of (your score earned minus 25 points) and 0 points.

Please be mindful of the deadlines, and start assignments early. Course staff will likely be less available after 5PM and during weekends, so plan accordingly if you need assistance.

# Problems [50 points]

## Problem 1 [6 points]

Using logical equivalence laws, show that:

**RUBRIC: 3 points if perfect (or only missed double negation). -1 per missing step**

a) $(p \lor q) \land \neg(\neg p \land q)$ is logically equivalent to p

**$\equiv (p \lor q) \land (\neg\neg p \lor \neg q)$ by De Morgan's**

**$\equiv (p \lor q) \land (p \lor \neg q)$ by Double negation**

**$\equiv p \land (q \lor \neg q)$ by Distributive law**

**$\equiv p \land T$ by negation law**

**$\equiv p$ by identity**

b) $\neg(x \lor \neg(x \land y))$ is logically equivalent to F (i.e. a contradiction).

**$\equiv (\neg x \land \neg\neg(x \land y))$ by De Morgan's**

**$\equiv (\neg x \land (x \land y))$ by Double negation**

**$\equiv ((\neg x \land x) \land y)$ by Associative law**

**$\equiv F \land y$ by negation law**

**$\equiv F$ by domination**

## Problem 2 [10 points]

Consider the following predicates:

**Student(x):** x is a student ← **Note:** This predicate was left in by mistake

**Smart(x):** x is smart

**Friends(x, y):** x and y are friends

**Plays(x, z):** x plays z

with the following domains:

domain of x and y: {all UB students}

domain of z: {sports played at UB}

Express each of the following statements using quantifiers, logical connectives, and the predicates defined above. You may use variables other than x, y, z as needed.

**Note**: Alice and Bob are UB students and frisbee is a sport played at UB.

**RUBRIC: 2 points if correct.**

a) There exists a smart student

**$\exists x$ Smart(x)**

b) Every student has at least one friend at UB

**$\forall x \exists y$ Friends(x, y)**

c) There is a student who is friends with every other student

**$\exists x \forall y$ Friends(x, y)**

d) Alice doesn't play frisbee

**$\neg$Plays(Alice, frisbee)**

e) Bob does not play any sports

**$\neg\exists x$ Plays(Bob, x) or $\forall x \neg$Plays(Bob, x)**

## Problem 3 [8 points]

Consider the following predicates:

**L(w):** Worker *w* is on a lunch break
**B(w):** Worker *w* is busy
**C(t):** Task *t* has been canceled
**T(t):** Task *t* is on the TODO list

Translate the following in to English:
**RUBRIC: 2 points if correct**

a) $\exists w\,(L(w) \land \neg B(w)) \rightarrow \exists t\, C(t)$
   **If a worker is on a lunch break and not busy, then a task has been canceled**
b) $\forall w\, B(w) \rightarrow \exists t\, T(t)$
   **If all workers are busy, then there's a task on the TODO list**
c) $\exists t\,(T(t) \lor C(t)) \rightarrow \exists w\, L(w)$
   **If a task on the TODO list or been canceled, then there's a worker on break**
d) $(\forall w\, B(w) \land \forall t\, T(t)) \rightarrow \exists t\, C(t)$
   **If all workers are busy, and all tasks are on the TODO list, then a task has been canceled**

## Problem 4 [6 points]

For each of the following statements, form the negation, then apply De Morgan's law, and/or the conditional law when applicable. Simplify the statements until the only negations that remain act on a single predicate, i.e. no negation should be outside of a quantifier or an expression involving logical operators. Show all steps.

**RUBRIC: 1 point for correct negation. 1 point for correct simplification. [1/2 if no work]**

a) $\forall x\,(P(x) \lor R(x))$
   **Negation:** $\neg\forall x\,(P(x) \lor R(x)) \equiv \exists x\,\neg(P(x) \lor R(x))$
   **Simplification:** $\equiv \exists x(\neg P(x) \land \neg R(x))$ **by De Morgan's Law**

b) $\forall y \exists z(P(y) \rightarrow \neg Q(z))$
   **Negation:** $\neg\forall y \exists z\,(P(y) \lor \neg Q(z)) \equiv \exists y \neg\exists z(P(y) \rightarrow \neg Q(z))$
   $\equiv \exists y \forall z \neg(P(y) \rightarrow \neg Q(z))$
   **Simplification** $\equiv \exists y \forall z \neg(\neg P(y) \lor \neg Q(z))$ **by implication law**
   $\equiv \exists y \forall z(\neg\neg P(y) \land \neg\neg Q(z))$ **by De Morgan's**
   $\equiv \exists y \forall z(P(y) \land Q(z))$ **by Double Negation**

c) $\exists x\,(P(x) \land (\forall z\,(\neg R(z) \rightarrow \neg Q(z))))$
   **Negation:** $\neg\exists x\,(P(x) \land (\forall z\,(\neg R(z) \rightarrow \neg Q(z)))) \equiv \forall x\,\neg(P(x) \land (\forall z\,(\neg R(z) \rightarrow \neg Q(z))))$
   **Simplification** $\equiv \forall x\,(\neg P(x) \lor \neg(\forall z\,(\neg R(z) \rightarrow \neg Q(z))))$ **by De Morgan's**
   $\equiv \forall x\,(\neg P(x) \lor (\exists z\,\neg(\neg R(z) \rightarrow \neg Q(z))))$ **by quantifier neg**
   $\equiv \forall x\,(\neg P(x) \lor (\exists z\,\neg(R(z) \lor \neg Q(z))))$ **by implication law**

$\equiv \forall x\, (\neg P(x) \lor (\exists z(\neg R(z) \land Q(z))))$ by De Morgan's

## Problem 5 [10 points]

Use the logical equivalence laws and/or logical reasoning laws to prove that each of the following arguments is valid.

**RUBRIC: 5 points each for perfect. -1 for each missing step.**

$$\begin{array}{c} a \to b \\ \neg c \to \neg b \\ \neg c \\ \hline \therefore \neg a \end{array} \qquad \begin{array}{c} \neg w \land x \\ \neg(w \lor y) \to z \\ y \to w \\ \hline \therefore z \end{array}$$

| | | |
|---|---|---|
| **1.** | $\neg c \to \neg b$ | **Hypothesis** |
| **2.** | $\neg c$ | **Hypothesis** |
| **3.** | $\neg b$ | **Mod. pon. 1,2** |
| **4.** | $a \to b$ | **Hypothesis** |
| **5.** | $\neg b \to \neg a$ | **Contrapos, 4** |
| **6.** | $\neg a$ | **Mod. pon. 3,5** |

| | | |
|---|---|---|
| **1.** | $\neg w \land x$ | **Hypothesis** |
| **2.** | $\neg w$ | **Simplification,1** |
| **3.** | $y \to w$ | **Hypothesis** |
| **4.** | $\neg w \to \neg y$ | **Contrapos., 3** |
| **5.** | $\neg y$ | **Mod. Pon. 2,4** |
| **6.** | $\neg w \land \neg y$ | **Conjunction, 2,5** |
| **7.** | $\neg(w \lor y)$ | **De Morgans, 6** |
| **8.** | $\neg(w \lor y) \to z$ | **Hypothesis** |
| **9.** | $z$ | **Mod Pon, 7,8** |

## Problem 6 [6 points]

Determine whether the following argument is valid. Justify your answer by first translating each statement into propositional logic to obtain the form of the argument. Then prove that the form is valid or not using equivalence laws and/or logical reasoning laws.

**RUBRIC: 3 points for argument, 3 points for counterexample (or prove validity of incorrect argument)**

The person is laughing if the comedian is good

The person is laughing

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∴ The comedian is good

**p: The person is laughing**
**q: The comedian is good**

$$\begin{array}{l} q \to p \\ p \\ \hline \therefore q \end{array}$$

**If q: F p: T then the hypotheses are T but the conclusion is F, therefore it is invalid**

# Problem 7 [4 points]

Prove the following statement by **contraposition**

**RUBRIC: 1 point for correct assumption, 2 points for derivation, 1 for correct conclusion**

*For every integer **x**, if **$3x^2$ - 7x + 1** is even, then **x** is odd*

**Assume x is even**

**Then exists integer i s.t. x = 2*i**
**By substitution we get $3*(2*i)^2 - 7*(2*i) + 1 = 12 * i^2 - 14 * i + 1 = 2 * (6i^2 -7i) + 1$**
**Since i is an integer, so is $6i^2 - 7i$**

**Therefore 3x2 - 7x + 1 is odd**
**Therefore we have proven the contrapositive is T, so the original statement is also T**

# Problem 8 (Extra Credit) [5 points]

Prove the following statement by **contradiction**

**RUBRIC: 1 point for correct assumption, 3 for derivation, 1 for identifying a contradiction**

*If **n** is an integer, and **$n^3$ - 9** is odd, then **n** is even*

**Let P(n): $n^3$ - 9 is odd, Q(n): n is even**

**Original Statement is $\forall n\ (P(n) \rightarrow Q(n))$ where domain is integers**
**Assume the opposite: $\neg \forall n\ (P(n) \rightarrow Q(n))$**

**$\equiv \exists n\ \neg(P(n) \rightarrow Q(n)) \equiv \exists n\ \neg(P(n) \rightarrow Q(n)) \equiv \exists n\ \neg(\neg P(n) \vee Q(n)) \equiv \exists n\ (P(n) \wedge \neg Q(n))$**

**Let x be an integer that satisfies our assumption.**
**Therefore x is odd (because $\neg$Q(x) must be TRUE)**
**Therefore there is an integer i s.t. x = 2i + 1**
**We also know that multiplying odd by odd is odd, so $x^3$ is therefore an odd number**

**By our assumption $x^3$ - 9 is odd (because P(x) must be TRUE).**
**An odd number minus an odd number is even, therefore $(x^3 - 9) - x^3 = 9$ is even.**
**But 9 is odd.**

**Therefore we have a contradiction, so our assumption must be false.**
**Therefore the original statement must be true.**