Homework #3

Due: 4/9/23 @ 11:59pm

Content Covered: Sets

Submission Instructions

Submit your completed homework to UBLearns electronically in PDF format. Any submissions that are not a PDF or not a legible PDF will not receive credit. We need to be able to read your submission to be able to grade your work. Your write-up should contain enough information from the problem so that a reader doesn't need to return to the text to know what the problem is (it is a good habit to rewrite each problem prior to solving it). There is no general rule for how much information from the problem to include, but it should be possible to read your homework and ascertain what the problem was and what your solution is accomplishing.

When writing up the solution, you may hand write the solutions and submit a scanned PDF, or write up the solutions electronically and convert them to a PDF. If you hand write your solutions, make sure that you write clearly and your writing is legible. Double check your scans to make sure that your scanned copy is legible. After you submit your work, make sure the file is visible. Download your submitted copy, open it, and see whether you submitted the correct file and your submitted file has not been corrupted during the upload.

You are able to upload your submission multiple times. Only the last file will be graded. Keep in mind that if your completed work consists of multiple pages and you submit a separate file for each page, only the last file submitted will be graded. In this case, only one page of your submission would be graded. **You are responsible for making sure that your submission goes through as intended.**

Your submitted work must be your own. Please review the course Academic Integrity Policy as outlined in the syllabus. **Failure to adhere to this policy will result in an F in the course.**

Late Policy

Late homework will be accepted up to 1 day late for a penalty of 25% of the total points. For example, if the homework is worth 100 points and you submit it one day late, you will receive the maximum of (your score earned minus 25 points) and 0 points.

Please be mindful of the deadlines, and start assignments early. Course staff will likely be less available after 5PM and during weekends, so plan accordingly if you need assistance.

Problems   [50 points]

Problem 1   [10 points]

Consider the following sets (let the universe be the set of all movies):

A: *The set of movies that are horror movies*
B: *The set of movies that star Nicolas Cage*

Describe each of the following in English:
RUBRIC: 2 points for a correct description

a) $A \cap B$ **The set of horror movies that star Nicolas Cage**

b) $A \cup B$ **The set of movies that are horror or star Nicolas Cage**

c) $A - B$ **The set of horror movies that don't star Nicolas Cage**

d) $B - A$ **The set of Nicolas Cage movies that are not horror movies**

e) $\overline{A}$ **The set of all movies that are not horror movies**

Problem 2   [10 points]

Consider the following sets (let the universe be $\mathbb{Z}$):

$A = \{ 2n \mid n \in \mathbb{Z} \}$
$B = \{ 3n \mid n \in \mathbb{Z} \}$
$C = \{ 4n \mid n \in \mathbb{Z} \}$
$D = \{ 6n \mid n \in \mathbb{Z} \}$
$E = \{ 8n \mid n \in \mathbb{Z} \}$

For each of the following, determine if it is TRUE or FALSE:
RUBRIC: 2 points if correct

a) $E \subseteq C \subseteq A$ **TRUE**

b) $A \subseteq C$ **FALSE**

c) $D \subseteq B$ **TRUE**

d) $\overline{D} \subseteq \overline{A}$ **FALSE**

e) $D = A \cap B$ **TRUE**

Problem 3   [6 points]

Consider sets **A**, **B**, and **C**, with the same universal set **U**. Draw the Venn Diagram corresponding to the following sets:

RUBRIC: 2 points if correct, 1 point if minor mistake. (solutions on last page)

    a) $(A \cap C) \cup B$

    b) $(B \oplus C) - A$

    c) $\overline{C} - (A \oplus B)$

Problem 4   [4 points]

Let $A_i = \{-i, i\}$.

Compute the following:

RUBRIC: 2 points if correct

    a) $\bigcup\limits_{i=1}^{\infty} A_i$ $\mathbb{Z}$ - {0}

    b) $\bigcap\limits_{i=1}^{\infty} A_i$ {}

Problem 5   [10 points]

Consider the following sets (**U** is the universal set for **A,B,C,D**):

$A = \{\, dog,\ cat,\ fish \,\}$
$B = \{\, dog,\ dog,\ fish,\ bear \,\}$
$C = \{\, dog,\ \{fish\},\ dog,\ \{dog,\ fish\},\ bear,\ \{\, fish,\ fish,\ dog,\ bear \,\} \,\}$
$D = \{\, cat,\ cat,\ cat,\ fish,\ fish,\ dog \,\}$

$U = \{\, dog,\ cat,\ fish,\ bear \,\}\ \cup\ \mathcal{P}(\{\, dog,\ cat,\ fish,\ bear\})$

Answer the following. Keep explanations to one sentence max.
RUBRIC: 0.5 point for correct answer, 0.5 point for explanation.

a) Is $B \subseteq C$? Explain.   **No. fish is in B but not in C.**

b) Is $D \subseteq A$? Explain.   **Yes. Everything in D is in A.**

c) Is $B \in A$? Explain.   **No. B is not an element of A.**

d) Is $B \in \mathcal{P}(C)$? Explain.   **No. B is not a subset of C, therefore not in P(C)**

RUBRIC: 1 points for correct answer, deduct 0.5 if minor mistake (at most 1 element off).

e) Write the contents of $A \cup C$ in roster notation.
{dog, cat, fish, bear, {fish}, {dog,fish}, {fish,fish,dog,bear}}

f) Write the contents of $A \cap C$ in roster notation.
{dog}

g) Write the contents of $B \times D$ in roster notation.
{(dog,cat),(dog,fish),(dog,dog),(fish,cat),(fish,fish),(fish,dog),(bear,cat),(bear,fish), (bear,dog)}

h) Write the contents of $\mathcal{P}(B)$ in roster notation.
{∅,{dog},{fish},{bear},{dog,fish},{dog,bear},{fish,bear},{dog,fish,bear}}

i) Write a partition of C with 2 sets (can be any sets you like).
{dog,bear} and {{fish}, {dog,fish}, {fish, fish, dog, bear}} for example.

RUBRIC: 0.5 point for correct answer, 0.5 point for explanation.

j) Does the list of sets: $\{dog\},\{dog\},\{fish\},\{bear\}$ partition B? Explain.
No. dog shows up in 2 sets of the partition.

Problem 6   [10 points]

Consider the alphabet $\{0, 1\}$. We have an interesting notation to define the universal set U in the context of strings. The universal set, U, of strings over the alphabet $\{0, 1\}$ is defined using the Kleene closure of the alphabet (denoted with the * operator):

$$U = \{0,1\}^* = \bigcup_{i \geq 0} \{0,1\}^i = \{0,1\}^0 \cup \{0,1\}^1 \cup \{0,1\}^2 \cup \{0,1\}^3 \cup \ldots$$

Let $A = \{0, 1, 01, 001\}$, with the universal set, U, of A defined above. Answer the following questions about A:

RUBRIC: 2 points if correct. 1 point for sets that are only 1 off of perfect.

    a) What is the shortest string in U?

        λ (or the empty string)

    b) How many strings are in $\overline{A}$?

        An infinite number

    c) Write the contents of $\{0,1\}^0 \cup \{0,1\}^1 \cup \{0,1\}^2$ in roster notation.

        {λ,0,1,00,01,10,11}

    d) Write the contents of $A \times A$ in roster notation. Since we are treating these elements as strings, they should be written as such. Your final answer should not include pairs (with parentheses and commas), and should not contain any duplicate strings.

        {00,01,001,0001,10,11,101,1001,010,011,0101,01001,0010,0011,00101,001001}

    e) Write the contents of $A \oplus (\{0,1\}^0 \cup \{0,1\}^1 \cup \{0,1\}^2)$

        {λ,00,10,11,001}

Problem 7 (Extra Credit)   [5 points]

For each of the following, state whether it is TRUE of FALSE. If it is TRUE, give your reasoning as to why. If it is FALSE, give a counterexample.

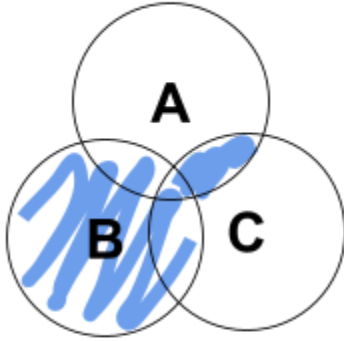**RUBRIC: 1 point if correct, 1 point for explanation**

    a) If $A \subseteq B$ and $B \in C$, then $A \in C$

        FALSE. A = {}, B = {1}, C = {{1},{2},{3}} as one possible counterexample

RUBRIC: 1 point if correct, 2 point for explanation

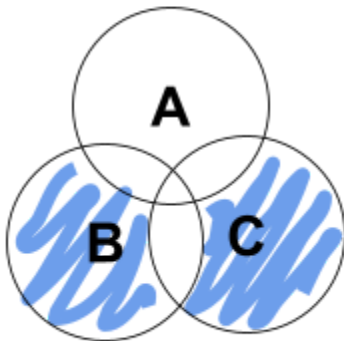   b) If $(A \cap C) = (B \cap C)$, then $A = B$

        FALSE. A = {1,2}, B = {1,3}, C = {1} as one possible counterexample

PROBLEM 3 SOLUTIONS

Problem 3a)

Problem 3b)

Problem 3c)