Homework #4

Due: 4/23/23 @ 11:59pm

Content Covered: Relations, Functions, Sequences, Recurrence Relations, Induction

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Submission Instructions

Submit your completed homework to UBLearns electronically in PDF format. Any submissions that are not a PDF or not a legible PDF will not receive credit. We need to be able to read your submission to be able to grade your work. Your write-up should contain enough information from the problem so that a reader doesn't need to return to the text to know what the problem is (it is a good habit to rewrite each problem prior to solving it). There is no general rule for how much information from the problem to include, but it should be possible to read your homework and ascertain what the problem was and what your solution is accomplishing.

When writing up the solution, you may hand write the solutions and submit a scanned PDF, or write up the solutions electronically and convert them to a PDF. If you hand write your solutions, make sure that you write clearly and your writing is legible. Double check your scans to make sure that your scanned copy is legible. After you submit your work, make sure the file is visible. Download your submitted copy, open it, and see whether you submitted the correct file and your submitted file has not been corrupted during the upload.

You are able to upload your submission multiple times. Only the last file will be graded. Keep in mind that if your completed work consists of multiple pages and you submit a separate file for each page, only the last file submitted will be graded. In this case, only one page of your submission would be graded. **You are responsible for making sure that your submission goes through as intended.**

Your submitted work must be your own. Please review the course Academic Integrity Policy as outlined in the syllabus. **Failure to adhere to this policy will result in an F in the course.**

Late Policy

Late homework will be accepted up to 1 day late for a penalty of 25% of the total points. For example, if the homework is worth 100 points and you submit it one day late, you will receive the maximum of (your score earned minus 25 points) and 0 points.

Please be mindful of the deadlines, and start assignments early. Course staff will likely be less available after 5PM and during weekends, so plan accordingly if you need assistance.

Problems   [50 points]

Problem 1   [10 points]

Consider the set **S** = {A, B, C, D}:

For each of the following relations on **S**, determine whether it is a partial order, a total order, an equivalence relation, or none of the above, and in one sentence explain why:

   a) {(A,B), (B,C), (C,D), (A,C), (B,D), (A,D)}

   b) {(A,A), (B,A), (B,B), (C,C), (C,D), (D,D)}

   c) {(A,A), (B,A), (A,B), (B,B), (C,C), (A,C), (C,A), (D,D)}

   d) {(A,A), (B,A), (A,B), (B,B), (C,C), (D,D)}

   e) {(A,A), (B,A), (A,B), (B,B), (C,C), (C,D), (D,D)}

Problem 2   [6 points]

For each of the following functions, determine whether it is injective, surjective, and/or invertible and explain why:

   a) Function $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is defined as $f((a, b)) = b + a$.

   b) Let **A = {1, 2, 4}** and function **sum:** $\mathscr{P}(A) \to$ **{0, 1, 2, 3, 4, 5, 6, 7}**, where **sum(X)** maps to the sum of all elements of **X**. **Reminder:** $\mathscr{P}(A)$ is the powerset of **A**.

   c) Function $f: \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = (x^2 + 1)/(x^2 + 2)$

Problem 3   [4 points]

Let set **G = { Monopoly, Catan, Deus, Orleans, Imhotep, Splendor, Clue }** and let the relation **R**: **G** $\to$ **G** be defined such that **a R b** iff **a = b** or the title of **a** is shorter than the title of **b**. For example, (**Clue**, **Catan**) is in **R** because **"Clue"** is shorter than **"Catan"**. (**G, R**) is a poset.

Draw the Hasse diagram for this partial ordering.

Problem 4   [10 points]

For each sequence below, (i) write out the first 5 terms, (ii) write out the explicit formula for the term with index n, and (iii) describe the sequence using a recurrence relation.

    a) $\{a_n\}$ is the geometric sequence with initial term a_0 = **-16** and common ratio ¼

    b) $\{b_n\}$ is the arithmetic sequence with initial term b_0 = **1999** and common difference 4

Problem 5   [10 points]

For each of the below recurrence relations, solve to find an explicit formula by using iteration. Show all work.

    a) a_0 = **0**, a_n = a_{n-1} + **2n**

    b) a_0 = **4**, a_n = ½ a_{n-1} + **2**

Problem 6   [10 points]

Let $\{a_n\}$ be the sequence defined by a_0 = **0**, a_1 = **3**, and a_n = **5**a_{n-1} - **4**a_{n-2} for **n ≥ 2**. Prove by strong mathematical induction that a_n = 4^n **− 1** for all **n ≥ 0**.

Your proof must include every part of the proof template referenced in class

Problem 7 (Extra Credit)   [5 points]

Answer each of the following:

    a) $\lfloor -2.1 \rfloor$ = ?

    b) $\lfloor$ ½ $+ \lceil$ ½ $\rceil \rfloor$ = ?

    c) $\lfloor a \rfloor$ = $\lceil a \rceil$ for all a ∈ ℝ. True or False? Give a counterexample if false.

    d) $\lfloor a \rfloor$ = $\lceil a \rceil$ - 1 = $\lceil a - 1 \rceil$ for all a ∈ R − ℤ. True or False? Give a counterexample if false.

    e) Is 40 congruent to 5 modulo 17? Why, or why not?