Homework #5

Due: 5/10/23 @ 11:59pm

Content Covered: Counting, Graphs

Submission Instructions

Submit your completed homework to UBLearns electronically in PDF format. Any submissions that are not a PDF or not a legible PDF will not receive credit. We need to be able to read your submission to be able to grade your work. Your write-up should contain enough information from the problem so that a reader doesn't need to return to the text to know what the problem is (it is a good habit to rewrite each problem prior to solving it). There is no general rule for how much information from the problem to include, but it should be possible to read your homework and ascertain what the problem was and what your solution is accomplishing.

When writing up the solution, you may hand write the solutions and submit a scanned PDF, or write up the solutions electronically and convert them to a PDF. If you hand write your solutions, make sure that you write clearly and your writing is legible. Double check your scans to make sure that your scanned copy is legible. After you submit your work, make sure the file is visible. Download your submitted copy, open it, and see whether you submitted the correct file and your submitted file has not been corrupted during the upload.

You are able to upload your submission multiple times. Only the last file will be graded. Keep in mind that if your completed work consists of multiple pages and you submit a separate file for each page, only the last file submitted will be graded. In this case, only one page of your submission would be graded. **You are responsible for making sure that your submission goes through as intended.**

Your submitted work must be your own. Please review the course Academic Integrity Policy as outlined in the syllabus. **Failure to adhere to this policy will result in an F in the course.**

Late Policy

Late homework will be accepted up to 1 day late for a penalty of 25% of the total points. For example, if the homework is worth 100 points and you submit it one day late, you will receive the maximum of (your score earned minus 25 points) and 0 points.

Please be mindful of the deadlines, and start assignments early. Course staff will likely be less available after 5PM and during weekends, so plan accordingly if you need assistance.

Problems [50 points]

Problem 1 [15 points]

Answer each of the following counting problems. For each problem you must explain how you got your answer. **Answers that show no work get no credit, even if the number is correct.**

Valid explanations should include things like which counting rules were applied (ie sum rule, product rule), and why order does or does not matter, when relevant, etc. For permutations and combinations, leave your answers in terms of $P(n, k)$ and $C(n, k)$.

a) Alice owns 3 dresses, 6 pairs of casual pants, 4 casual shirts, 2 pairs of fancy pants, and 3 fancy shirts. If she does not mix fancy and casual, how many outfits can she make that include either a dress, or shirt and pants?

b) How many possible poker hands can you make from a standard deck of playing cards? (A poker hand is a selection of 5 cards from a deck of 52 unique cards)

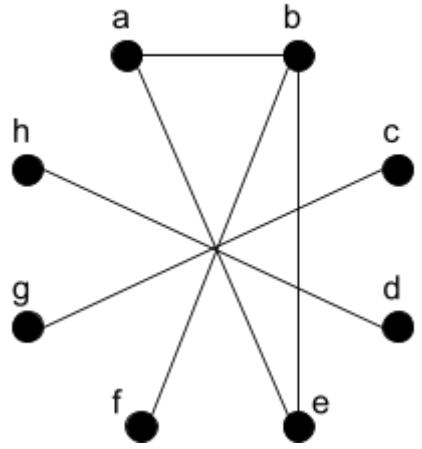
c) A sequence of DNA can be represented as a sequence of bases, where the possible bases are A, C, G, T. How many DNA sequences of length 5 that do not contain the sequence TAG exist?

d) A collectible trading card game sells packs that contain 10 random cards each. If there are 150 possible cards, how many packs must you buy to **guarantee** you get at least 4 of the same card?

e) Suppose we have everyone in the room rank their top 5 Star Wars movies in order from favorite to least favorite (there are 9 Star Wars movies to choose from). How many people must be in the room to **guarantee** that two people in the room have the same exact ranking?

Problem 2 [10 points]

Let $G_1 = (V_1, E_1)$ be the following graph:



a) Write out E_1 in roster notation.

b) What is the total degree of G_1?

c) Write out the connected components of G_1.

d) Do the connected components partition V_1?

e) Write out the neighborhood of $\{e,f\}$.

Problem 3 [10 points]

Let $G_2 = (V_2, E_2)$ be the directed graph represented by the following adjacency matrix:

	1	2	3	4
1	0	0	1	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	0

a) Draw G_2.

b) What is $|V_2|$? What is $|E_2|$?

c) Is G_2 a simple graph? Why or why not?

d) Which vertex has the largest out-degree? The largest in-degree?

e) A directed graph is weakly connected if replacing every directed edge with an undirected edge results in a connected graph. A directed graph is strongly connected if for every pair of vertices (x,y) there is a directed path from x to y (and vice-versa). Is G_2 strongly connected, weakly connected, or neither?

Problem 4 [6 points]

A degree sequence of a graph is the sequence of degrees of the vertices of the graph in non-increasing order. For example, the degree sequence of graph G_1 from Problem 1 is 3, 2, 2, 1, 1, 1, 1, 1. An arbitrary sequence is **graphic** if it can be the degree sequence of a simple graph. For each of the following sequences, state whether they are graphic or not. If they are not graphic, state why. If they are graphic, draw a **connected** graph with that degree sequence.

a) 5, 3, 2, 2, 2, 2

b) 4, 3, 2, 1, 1

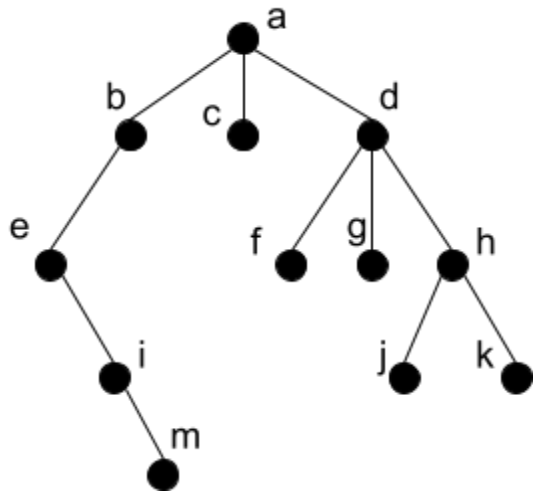
c) 6, 4, 4, 2, 2, 2, 2

Problem 5 [4 points]

For each **graphic** sequence in problem 4 (ignore any that were non-graphic), state whether or not they have an Euler circuit. If it does not, state why. If it does, write out a possible Euler circuit (label your vertices with letters a,b,c,...).

Problem 6 [5 points]

Let ***T*** be the following rooted tree.



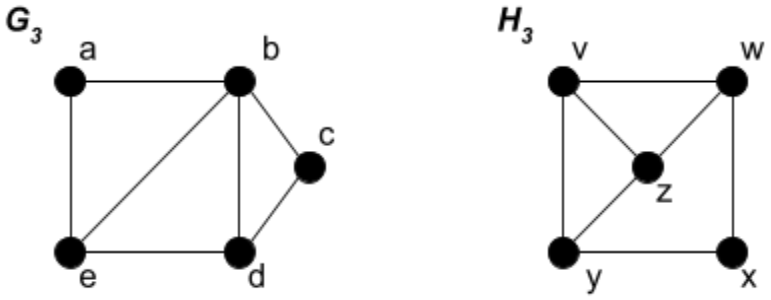
a) What is the height of ***T***?

b) What are the leaves of ***T***?

c) What are the descendents of ***d***?

Problem 7 (Extra Credit) [5 points]

Let G_3 and H_3 be the following graphs:



a) Both of the above graphs can be assigned a valid 3-coloring. List out the five vertices for each graph and assign each one a color from {red, blue, yellow} that results in a valid 3-coloring.

b) Are G_3 and H_3 isomorphic? If so, provide the isomorphism. If not, state why.