CSE 191 - Midterm Exam - Version 1

3/8/23 @ 9AM, Talbert 107

Name: ______________________________ Person #: ________________________

UBIT: ______________________________ Seat #: ________________________

Academic Integrity

My signature on this cover sheet indicates that I agree to abide by the academic integrity policies of this course, the department, and university, and that this exam is my own work.

Signature: ____________________________________ Date: ____________________________

Instructions

1. This exam contains ~~9 total pages~~ (including this cover sheet). Be sure you have all the pages before you begin.
2. Clearly write your name, UBIT name, person number, and seat number above. **Additionally, write your UBIT name at the top of every page now.**
3. You have 1 hour and 20 minutes to complete this exam. Show all work where appropriate, but keep your answers concise and to the point.
4. After completing the exam, sign the academic integrity statement above. Be prepared to present your UB card upon submission of the exam paper.
5. You must turn in all of your work. No part of this exam booklet may leave the classroom.

DO NOT WRITE BELOW

P1	P2	P3	P4	(P5)	Total
6	7	12	5	(4)	30

RUBRIC FOR ALL MULTIPLE ANSWER:
2 points if perfect
1 point if off by one (either added one wrong answer or missed on right answer)

RUBRIC FOR ALL MULTIPLE CHOICE:
2 points if correct, 0 otherwise

Part 1 - Propositional Logic [6 Points]

1. Let p = T, q = F, r = T. [2 points]
   Select **ALL** expressions that evaluate to TRUE:

   a) $q \rightarrow ((p \land \neg q) \lor r)$

   b) $(\neg r \lor q) \leftrightarrow (\neg p \land q)$

   c) $\neg q \oplus r$

   d) $\neg q \lor r$

   e) $(r \land p) \rightarrow q$

2. Let the propositional variables p, q, and r represent the propositions: [2 points]

   p: You won the game
   q: You scored the most points
   r: You had the most fun

   Select the logical expression that represents the statement:
   "To win the game, it is necessary that you score the most points"

   a) $p \leftrightarrow q$

   b) $q \rightarrow p$

   c) $r \rightarrow p$

   d) $p \rightarrow q$

   e) $p \rightarrow (q \land r)$

   f) None of the above

3. From the expressions below, select **ALL** of the tautologies: [2 points]

   a) **$F \rightarrow (p \land q)$**

   b) $(F \rightarrow (p \land q)) \rightarrow (p \land q)$

   c) $r \leftrightarrow \neg r$

   d) **$(p \oplus q) \rightarrow (p \lor q)$**

   e) $(p \lor q) \rightarrow (p \oplus q)$

   f) $(p \land q) \land (p \oplus q)$

Part 2 - Predicates and Quantifiers [7 Points]

For all questions in this part, consider the following predicates:

$P(x)$: x accommodates 4 players
$Q(x)$: x is cooperative
$R(x)$: x has purple as a player color
$S(x)$: x takes less than an hour to play

Where the domain of x is { all board games I own }

4. Write the logical expression that is equivalent to: [3 points]
"None of my cooperative board games accommodate 4 players"
RUBRIC: **3 points for perfect**
2 points if only mistake was switched → and ∧
1 point if only mistake was wrong quantifier/negation

$\neg\exists x(Q(x) \wedge P(x))$ or $\forall x(Q(x) \rightarrow \neg P(x))$

5. Select the logical expression that is equivalent to: [2 points]
"Every game I own that doesn't accommodate 4, can be played in under an hour"

   a) $\forall x(\neg P(x) \vee S(x))$

   b) $\forall x\,(\neg P(x) \rightarrow S(x))$

   c) $\neg\forall x\,(P(x) \rightarrow S(x))$

   d) $\neg\exists x\,(P(x) \wedge S(x))$

   e) $\forall x\,(\neg P(x) \wedge S(x))$

6. From the expressions below, select **ALL** that have free variables: [2 points]

   a) $\neg\forall x\, P(x) \rightarrow S(x)$

   b) $\forall x\, Q(x) \leftrightarrow \exists y\, R(y)$

   c) $\exists x,y\,(P(x) \vee P(y))$

   d) $\exists x, \forall y\,(P(y) \rightarrow P(x))$

   e) $\exists x\, S(x) \rightarrow \forall y\,(R(x) \oplus P(y))$

Part 3 - Logical Equivalence, Reasoning, and Proofs [12 Points]

7. Select the logical expression that is equivalent to: [2 points]
   $\neg(p \land q) \land (\neg p \lor r)$

   a) $\neg(p \land q \land \neg r)$

   b) $p \lor (q \land r)$

   c) $p \land (\neg q \lor \neg r)$

   d) $p \lor q \lor r$

   e) $\underline{\neg p \lor (\neg q \land r)}$

8. Find the assignment of truth values to p and q that prove that argument below is invalid. [2 points]
   RUBRIC: 2 points if perfect, 0 otherwise
   $p \rightarrow q$ **p: F, q: F**
   $q \rightarrow p$
   ———
   $\therefore q$

9. Use logical inference and/or logical equivalence rules to prove that the following argument is valid: [3 points]

   If I've made it this far, then I've conquered the enemies and found the treasure
   I haven't found the treasure
   ————————————————————————————
   ∴ I haven't made it this far
   RUBRIC: 1 point for translating to propositions, 1 point for proof structure
   1 for correct steps/justifications [award all 3 if this part is perfect]

   p: I've made it this far
   q: I've conquered the enemies
   r: I've found the treasure

   $p \rightarrow (q \land r)$
   $\neg r$
   ———
   $\therefore \neg p$

   1. **$\neg r$** **Hypothesis**
   2. **$\neg r \lor \neg q$** **Addition, 1**
   3. **$\neg(r \land q)$** **De Morgan's 2**
   4. **$p \rightarrow (r \land q)$** **Hypothesis**
   5. **$\neg(r \land q) \rightarrow \neg p$** **Contrapositive, 4**
   6. **$\neg p$** **Modus ponens, 3, 5**

10. Consider the following statement: [2 points]
    If x < 10 and y ≥ 0 then x < y + 10

    If we wanted to prove this statement with a **proof by contraposition**, what assumption would we begin our proof with?

    a) x ≥ 10 or y < 0

    b) x < y + 10

    c) x ≥ y + 10

    d) x < 10 and y ≥ 0

    e) x < 10 and y ≥ 0 and x < y + 10

11. Use a direct proof to show that for all integers ***x***, ***y***, and ***z***, if ***y*** divides ***x*** and ***z*** divides ***y***, then ***z*** divides ***x***. **Note:** If ***a*** divides ***b***, then we can write ***b*** = ***an*** where ***n*** is an integer. [3 points]

    RUBRIC: **1 point for correct starting assumption**
    1 point for correct steps
    1 point for correct conclusion

    Assume y divides x and z divides y

    (1) Then exists an integer n s.t. x = y * n
    (2) There exists an integer m s.t. y = z * m

    Then we can substitute (2) into (1) to get
    (3) x = z * n * m

    Since n and m are integers, m * n is also an integer

    Therefore z divides x

Part 4 - Sets [5 Points]

12. Select **ALL** expressions that evaluate to TRUE: [2 points]

a) $\{ \varnothing \} \subseteq \{ \varnothing \}$

b) $\{4\} \in \{4\}$

c) $2 \in \{1, 2, 3\} \cup \{4, 5, 6\}$

d) $\{2, 4, 6\} \subseteq \{2, 4, 6, 8, 10\} \cap \{2, 4, 8, 16, 32\}$

e) $12 \in \{2, 12, 22, 32\} - \{0, 12, 144\}$

13. Consider the following set: $\boldsymbol{S} = (\mathbb{Z}^+ \cap \{ x \mid x < 10 \}) - \{ x \mid x \text{ is odd} \}$. [3 points]
Write ***S*** using roster notation.

RUBRIC: **1 point if contains only integers < 10**
2 point if it contains only even integers

***S* = { 2, 4, 6, 8 }**

Part 5 - Extra Credit [5 Points]

14. Consider the following statement: [2 points]
    If the product of two integers (a)(b) is odd, then a is odd and b is odd

    A **proof by contradiction** for this statement would start with which assumption:

    a) The product of two integers **(a)(b)** is odd

    b) **a** is even or **b** is even

    c) The product of two integers **(a)(b)** is even

    d) <u>The product of two integers (a)(b) is odd and a is even or b is even</u>

    e) **a** and **b** are not odd

15. Translate the following sentence into symbolic logic using predicates and quantifiers [3 points]
    There exists a real number y such that for every real number x, y + x = x
    RUBRIC: **1 point for mentioning domain**
    1 point for including existential and universal quantifiers
    1 point if they are in the right order

    Domain is all real numbers
    $\exists y, \forall x \ (y + x = x)$

Scrap Paper