CSE 191 Recitation

2/13/23 - 2/17/23 - Logical Equivalence

Special Cases

A **tautology** is a compound proposition that is ALWAYS TRUE

- Come up with some examples

A **contradiction** is a compound proposition that is ALWAYS FALSE

- Come up with some examples

A **contingency** is a compound proposition is neither

- Try to modify one of the previous examples to turn it into a contingency

Logical Equivalence

Two propositions, $\boldsymbol{p}$ and $\boldsymbol{q}$ are **logically equivalent** if $\boldsymbol{p} \Leftrightarrow \boldsymbol{q}$ is a tautology

We can also say they are equivalent if their truth values match in every row of their truth tables. Why?

Examples

Show that the following two statements are logically equivalent

"If I like apples then I like oranges"

"I don't like apples or I like oranges"

Examples

Show that

$$(p \land q) \rightarrow (r \lor s) \equiv s \lor r \lor \neg q \lor \neg p$$

by constructing a truth table

Examples

Show that

$$(p \wedge q) \rightarrow (r \vee s) \equiv s \vee r \vee \neg q \vee \neg p$$

by using laws of equivalence

Equivalence		Name
$p \land T \equiv p$	$p \lor F \equiv p$	Identity laws
$p \lor T \equiv T$	$p \land F \equiv F$	Domination laws
$p \lor p \equiv p$	$p \land p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$		Double negation law
$p \lor q \equiv q \lor p$	$p \land q \equiv q \land p$	Commutative laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$	$(p \land q) \land r \equiv p \land (q \land r)$	Associative laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive laws
$\neg(p \lor q) \equiv \neg p \land \neg q$	$\neg(p \land q) \equiv \neg p \lor \neg q$	De Morgan's laws
$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$	Absorption laws
$p \lor \neg p \equiv T$	$p \land \neg p \equiv F$	Negation laws

$p \to q \equiv \neg p \lor q$
$p \to q \equiv \neg q \to \neg p$
$p \lor q \equiv \neg p \to q$
$p \land q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \land \neg q$
$(p \to q) \land (p \to r) \equiv p \to (q \land r)$
$(p \to q) \land (q \to r) \equiv (p \lor q) \to r$
$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
$(p \to q) \lor (q \to r) \equiv (p \land q) \to r$

$p \Leftrightarrow q \equiv (p \to q) \land (q \to p)$
$p \Leftrightarrow q \equiv q \Leftrightarrow p$
$p \Leftrightarrow q \equiv \neg p \Leftrightarrow \neg q$
$p \Leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
$\neg(p \Leftrightarrow q) \equiv p \Leftrightarrow \neg q$