CSE 191 Recitation

Arguments

An **argument** is a list of propositions called *hypotheses* and a single proposition called the *conclusion*

An argument is **valid** if \((p_1 \land p_2 \land p_n) \rightarrow c\) is a tautology

Recall the truth table for implication

- The only time an implication is F is if the premise is T, and the conclusion is F
- So an argument is **invalid** if it is possible for all the hypotheses to be T, and the conclusion be F

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>(p \rightarrow q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
Invalid Arguments

For the following arguments, come up with counterexamples to show they are invalid.

\[ a \rightarrow b \]

\[ p \land q \]

\[ q \lor \neg r \]

\[ r \rightarrow \neg p \]

\[ x \lor y \lor z \]

\[ x \land z \]

\[ \therefore r \land q \]

\[ \therefore \neg y \]
Logical Reasoning Proofs

Translate the following statements to a formal argument, then prove validity via truth table, and with a proof.

If I am with friends, I am playing a game

If I am playing a game, I am happy

I am with friends

∴ I am happy
Mathematical Proof Examples

Proof by Cases: Prove that if \( n \) is an integer, then \( n^2 \geq n \)

What are your exhaustive cases? Prove each case? Note: you can assume we've proven that \( n^2 \geq 0 \)

Proof by Contraposition: Prove that for any integers \( x \) and \( y \), if both \( x + y \) and \( xy \) are even, then both \( x \) and \( y \) are even.

What implication are you trying to prove? What is its contrapositive? What is the starting assumption of your proof?