CSE 191 Recitation

4/24/23 - 4/28/23 - Graphs

Graph Examples

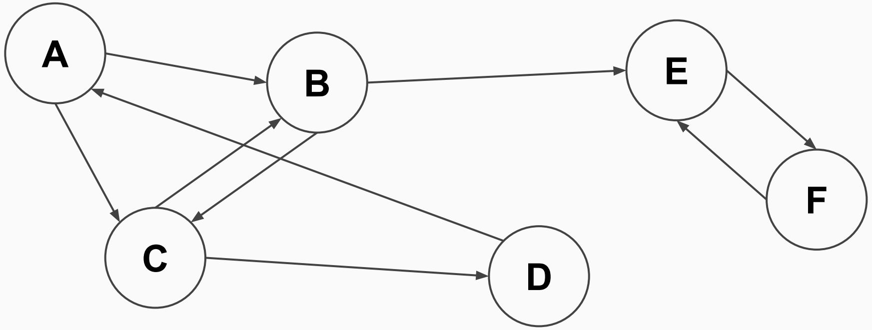
For the following situations, how could we represent them as graphs?

(what would vertices represent, what would edges represent, etc?)

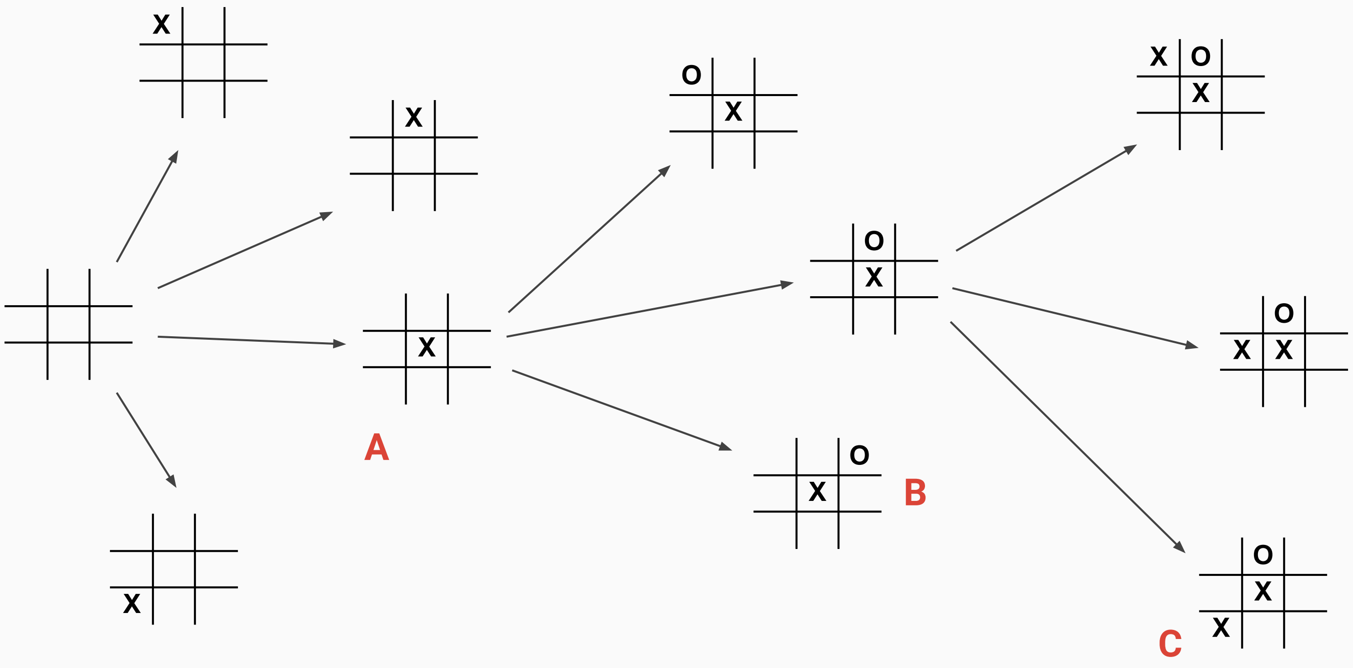
1. A map of the city of Buffalo
2. Twitter
3. A game of Tic-Tac-Toe

Graph Representation

Write out the Adjacency List and Adjacency Matrix representations of the following graph
How much "space" (number of characters you had to write) does each require?
Verify your representations by checking the out degree/in degree of each node



Tic Tac Toe Example



Note: This does not show all edges / vertices…

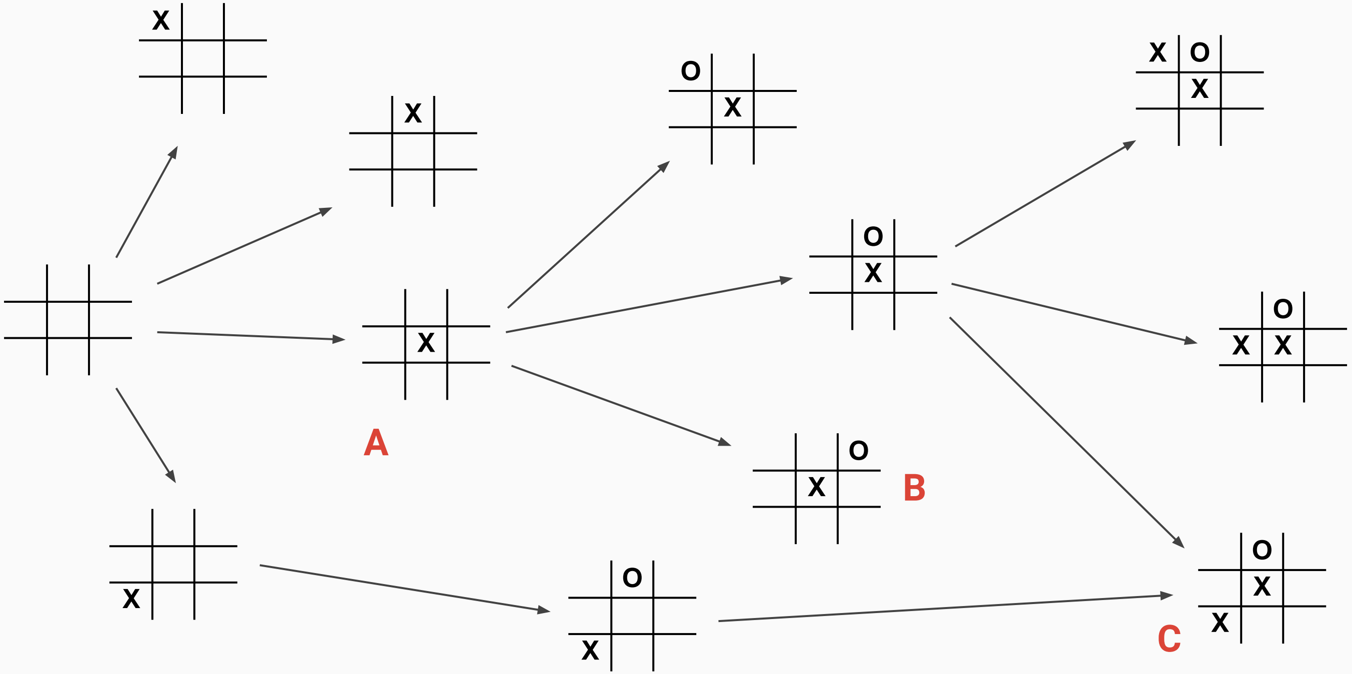
What is the out degree of the vertex for the empty board? What about the in degree?

What is the out degree of the vertex labeled A? B?

How many edges are in the full graph?

Is the in degree of every non-starting node 1?

Tic Tac Toe Example



Note: This does not show all edges / vertices…

What is the out degree of the vertex for the empty board? 9
What about the in degree? 0

What is the out degree of the vertex labeled A? B? 8, 7

How many edges are in the full graph? 9!

Is the in degree of every non-starting node 1? No ie C

Handshake Theorem

Use a proof by contradiction, the handshake theorem, and the pigeonhole principle to show that a **connected**, **simple**, **undirected** graph with 8 vertices and 23 edges cannot have a vertex with degree 1.

Handshake Theorem

Proof:
Assume there is a vertex with degree 1.

This vertex must be incident on exactly 1 edge. Remove this vertex and this edge from the graph. We now have a connected, simple, undirected, graph with 7 vertices and 22 edges.

By Handshake theorem: 2*22 = 44 = deg(v1) + deg(v2) + … + deg(v7). We have 7 holes, and 44 pigeons.

By generalized pigeonhole principle: 44/7 = 6.28, therefore degree of at least one vertex is at least 7.

The vertex that has degree ≥ 7 either must have a loop, or has 7 edges to the 6 other vertices. By pigeonhole this means there must be at least 2 edges to the same vertex, so the graph is not simple.

This is a contradiction (we said the graph is simple, derived that it is not simple). Since our original assumption led to a contradiction, our original assumption must be false.